Games and Economic Behavior ••• (••••) •••–•••

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Approximation in mechanism design with interdependent values ☆

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ARTICLE INFO

Article history:
Received 28 January 2014
Available online xxxx

JEL classification:
D44
D82

Keywords:
Mechanism design
Approximation
Interdependent values
Revenue maximization

ABSTRACT

This paper studies the revenue maximization problem in environments wherein buyers have interdependent values and correlated types. We show that (1) when the system of feasible sets is a matroid and buyer valuations satisfy a single-crossing condition, the generalized Vickrey–Clarke–Groves mechanisms with lazy reserves (VCG-L) are ex-post incentive compatible and ex-post individually rational; (2) if, in addition, the valuation distribution satisfies a generalized monotone hazard rate condition, the VCG-L mechanism with conditional monopoly reserves is approximately optimal. Then we construct an ascending auction that implements the truth-telling equilibrium of a VCG-L mechanism in ex-post equilibrium. Finally, we discuss the connection between the VCG-L mechanisms and greedy algorithms studied in Lehmann et al. (2002) and deferred-acceptance auctions studied in Milgrom and Segal (2014), and the impact of competition by proving a Bulow and Klemperer (1996) type result.

1. Introduction

In the independent private values (IPV) setting, it is well known that the Vickrey–Clarke–Groves (VCG) mechanism is ex-post incentive compatible[1] and ex-post individually rational. In addition, when there is a single item for sale and the agent values are drawn from identical distributions, the seminal work of Myerson (1981) shows that the VCG mechanism with a reserve price is revenue-maximizing in the "regular" case with increasing virtual valuation functions. Recent work demonstrates that this idea is more general. For example, Hartline and Roughgarden (2009) show that in a variety of IPV settings the VCG mechanism with buyer specific reserves is "approximately" optimal. This paper considers the design of approximately optimal mechanisms for the case of interdependent values, which is an important generalization of the above setting.

The case of interdependent values and correlated types is pertinent to many practical applications and has received much attention in the literature on auctions, commencing with Milgrom and Weber (1982). In an interdependent values setting, one buyer's valuation for winning can depend on other buyers' private information. In addition, the private information of buyers may be correlated. This paper considers a setting where the buyer's private information can be summarized by a one-dimensional signal. As a motivating example, suppose that the item for sale may be resold, and that buyers have different information about future states of the world, e.g. market conditions. Then the information possessed by other

☆ A one page abstract of an earlier version of this paper appears in the Proceedings of the EC' 13 Conference.

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[1] In the case of private values, ex-post incentive compatibility is equivalent to dominant strategy incentive compatibility for direct mechanisms.

http://dx.doi.org/10.1016/j.geb.2016.01.006
0899-8256/

Please cite this article in press as: Li, Y. Approximation in mechanism design with interdependent values. Games Econ. Behav. (2016), http://dx.doi.org/10.1016/j.geb.2016.01.006

buyers, if known to a particular buyer, may affect his valuation of winning. For example, a buyer's value of an art piece depends not just on his private consumption value, but also on his beliefs about the art's resale value, which depends on other buyers' values. Furthermore, in many applications buyers may have differing but correlated private information. A classic example is that of an oil tract that is for sale, where each potential buyer surveys the tract and estimates the extractable oil — buyers' estimates are therefore statistically correlated. Additionally, a buyer's private information will impact others' valuations — for example, if buyer 1 finds that buyer 2 has a lower estimate on the amount of extractable oil, he may revise his valuation downward. The importance of interdependent values in auctions has been empirically studied (see, e.g. Hendricks and Porter, 1988 and Hendricks et al., 2003).

Instead of considering a single item for sale, we consider an environment where there is a system of feasible sets of buyers specifying which of them can win simultaneously. For example, in the k-unit auction with unit-demand buyers, the seller can sell to at most k buyers. The feasible sets are precisely those subsets that contain no more than k buyers. Another example is a combinatorial auction with single-minded buyers; that is, each buyer cares only about a specific bundle. In this case, a feasible set corresponds to a subset of buyers seeking mutually disjoint bundles. In such environments, identifying the optimal mechanism remains an open question. This paper investigates a simpler question: can we find a simple mechanism that is ex-post incentive compatible and which performs "reasonably" well?

Specifically, we study the performance of the *VCG mechanisms with lazy reserves* or the VCG-L mechanisms. A VCG-L mechanism (see, e.g., Dhangwatnotai et al., 2010) (1) first runs the VCG mechanism to select the tentative winners, and (2) then removes the tentative winners whose types are below their reserves. Another variation of the VCG mechanisms with reserves is the *VCG mechanisms with eager reserves* or the VCG-E mechanisms (see, e.g., Hartline and Roughgarden, 2009). In a VCG-E mechanism, steps (1) and (2) are reversed.[2] We focus on the VCG-L mechanisms rather than the VCG-E mechanisms in this paper since, as we argue below, the VCG-E mechanisms are generally not ex-post incentive compatible in the interdependent values setting.

As is standard in the literature studying ex-post incentive compatibility for interdependent values problems, we assume that agent valuation satisfies a *single-crossing condition*. Under this condition, the VCG mechanism is ex-post incentive compatible in single-item auctions (see, e.g. Ausubel, 2000). Unfortunately, this is not necessarily true once we leave the single-item auction setting. In Section 3.1, we exhibit a novel example in which the VCG mechanism is not ex-post incentive compatible. The first contribution of this paper is to identify the environment — the system of feasible sets that form a *matroid* (described below) — in which the VCG mechanism is still ex-post incentive compatible.

The matroid setting covers many interesting economic applications. Examples include single-item auctions, the allocation of homogeneous goods (Ausubel, 2004), digital good auctions (Goldberg et al., 2001), scheduling matroid (Demange et al., 1986) and pairwise kidney exchange (Roth et al., 2005), among others.[3] It is also well known that the matroid feasibility constraint implies good properties of mechanisms in the IPV setting, such as the revenue guarantees of VCG mechanisms with buyer specific reserves (Hartline and Roughgarden, 2009), and the existence of ascending implementations (Bikhchandani et al., 2011). This paper, for the first time, demonstrates that the matroid feasibility constraint also implies good properties of mechanisms in the interdependent values setting.[4]

Clearly, if the VCG mechanism is ex-post incentive compatible, then the VCG-L mechanisms are ex-post incentive compatible. Furthermore, the VCG-L mechanisms are ex-post individually rational. The same is not true, however, for the VCG-E mechanisms. In Section 3.1, we provide an example of a singe-item auction in which the VCG-E mechanisms are not ex-post incentive compatible.

We call the VCG-L mechanism with *conditional monopoly reserves the VCG-L* mechanism*.[5] We prove that if, in addition, the buyer valuations and the distribution of their private information satisfy a generalized monotone hazard rate condition, the expected revenue generated by the VCG-L* mechanism is at least $1/e$ ($e \approx 2.718$) of the optimal revenue achievable by any Bayesian incentive compatible and interim individually rational mechanism, where e is the basis of the natural logarithm. The proof uses the fact that distributions meeting the monotone hazard rate condition have tails no heavier than that of an exponential distribution (which has a constant hazard rate). The bound e is tight, even in comparison to the optimal revenue achieved by ex-post incentive compatible and ex-post individually rational mechanisms. We also show that the generalized monotone hazard rate condition is necessary for this result. Even for settings satisfying Myerson's regularity condition, no constant approximation factor is possible.

Then, we construct an ascending auction that implements the truth-telling equilibrium of a VCG-L mechanism in ex-post equilibrium. We believe that ascending auctions are important for the following reasons. First, one of the major disadvantages of the VCG mechanism in the interdependent values setting is that it is not "detail free" — the seller is required to know the fine details of buyer valuation functions, as well as the joint distribution of their private information.[6] Without reserve prices, our ascending auction is "detail free". However, we do need buyer valuation functions to be common knowledge among them in order to ensure that the buyers can calculate equilibrium. This assumption about the buyers'

[2] See Dhangwatnotai et al. (2010) and Chawla et al. (2014) for comparisons between the VCG-L and VCG-E mechanisms in the IPV settings.

[3] See Bikhchandani et al. (2011) for more on matroid applications.

[4] I thank the AE for suggesting this argument.

[5] Since the monopoly reserves could depend on other buyers' reports, we call them *conditional monopoly reserves*.

[6] See Wilson (1987), Krishna (2009), Milgrom (2004) and Klemperer (2004) for more discussions on detail-free auction design.

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