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Choosing k from m: Feasible elimination procedures reconsidered ☆

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We show that feasible elimination procedures (Peleg, 1978) can be used to select k from m alternatives. An important advantage of this method is the core property: no coalition can guarantee an outcome that is preferred by all its members. We also show that the problem of determining whether a specific k-tuple can result from a feasible elimination procedure is computationally equivalent to the problem of finding a maximal matching in a bipartite graph.

1. Introduction

This paper focuses on the question of how to choose a set of k alternatives from a set of $m > k$ alternatives, by aggregating the preferences of voters over alternatives in a way that cannot be manipulated in the following sense: no coalition of voters, by possibly misrepresenting their preferences, can guarantee an outcome that all members of the coalition prefer to the outcome obtained by sincere, truthful voting. For instance, the set of k alternatives is a department board, which has to be composed from a set of available candidates, and we are looking for a voting method such that no coalition of voters, by not voting sincerely, can guarantee a board that all members of the coalition prefer. In order to achieve this, we will use an extension of a method that was originally proposed by Peleg (1978) as a reaction to the Gibbard–Satterthwaite Theorem.

The Gibbard–Satterthwaite Theorem (Gibbard, 1973, and Satterthwaite, 1975) states that every non-dictatorial social choice function whose range contains at least three alternatives, is manipulable: there exists a profile of preferences and a voter who, by deviating and reporting a preference different from his true one, can obtain an outcome which he prefers, according to his true preference, to the sincere outcome. From a game-theoretic point of view, a social choice function combined with a (true) preference profile is a game in strategic form, the strategy set of each player being the set of preferences over the alternatives; outcomes of the game are evaluated according to the true preferences. Peleg (1978) showed that there are reasonable (anonymous, Maskin monotonic) social choice functions such that the resulting game has a strong Nash equilibrium resulting in the sincere outcome whatever the true preferences. Social choice functions with this

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property are called exactly and strongly consistent (ESC). In particular, for an ESC social choice function the sincere outcome is in the core of the relevant voting game: this means that there is no coalition of voters that has a strategy profile which guarantees an alternative that all members of the coalition prefer over the sincere outcome. Of course, the sincere outcome may not be the unique outcome of a strong Nash equilibrium.[1]

The central tool, introduced in Peleg (1978), to obtain these social choice functions is the concept of a feasible elimination procedure. In such a procedure, applied to a profile of preferences, alternatives are eliminated one by one, until a final alternative remains: this is called a maximal alternative. In this paper we use this procedure to select $k > 1$ alternatives, simply by taking the last k instead of only the last alternative. We show that for at least two extensions of voters' preferences over the alternatives to (ordered) k-tuples of alternatives, this method has the core property: no coalition can guarantee an outcome (k-tuple) that is preferred by all its members. Formally, this core is defined as the core of the effectivity function induced by this method. We show, by an example, that some well-known existing methods (single transferable vote, plurality, plurality with run-off) violate the core property.

We also show that the problem of determining whether a specific k-tuple can result from a feasible elimination procedure is computationally equivalent to the problem of finding a maximal matching in a bipartite graph. The latter problem can be solved in polynomial time (Hopcroft and Karp, 1973).

Section 2 introduces feasible elimination procedures and Section 3 shows that these procedures provide a method to choose k alternatives from a set of m alternatives such that no coalition of voters can guarantee a better result by manipulation. We also show that this method is Maskin monotonic. Section 4 considers the computational aspect, and Section 5 concludes.

Notations. The following basic notations are used throughout. For a set D, $|D|$ denotes the cardinality of D, $P(D)$ the power set, i.e., the set of all subsets of D, and $P_0(D)$ the set of all nonempty subsets of D.

2. Preliminaries

Let A be a set of m *alternatives*, $m \geq 2$, and let $N = \{1, \ldots, n\}$, $n \geq 2$, be a set of *voters*. Denote by L the set of all linear orderings, i.e., complete, antisymmetric and transitive binary relations, of A. An element of L is also called a *preference*, and an element of L^N a *preference profile*. If xR^iy for some $x, y \in A$ and $i \in N$, where $R^i \in L$, then this is interpreted as voter i strictly preferring x over y.

A *social choice correspondence* (SCC) is a function $H : L^N \to P_0(A)$. An SCC H is *anonymous* if for all $R^N \in L^N$ and for all permutations π of N, $H(R^1, \ldots, R^n) = H(R^{\pi(1)}, \ldots, R^{\pi(n)})$. It is *Paretian* if for all $x, y \in A$ and $R^N \in L^N$, if $x \neq y$ and yR^ix for all $i \in N$, then $x \notin H(R^N)$. It is *Maskin monotonic* (Maskin, 1999) if it satisfies the following. Let $R^N, Q^N \in L^N$ and let $x \in H(Q^N)$. If xQ^iy implies xR^iy for all $y \in A$ and $i \in N$, then $x \in H(R^N)$.

In this paper we are especially interested in the SCC derived from feasible elimination procedures, introduced by Peleg (1978).

Definition 2.1. Assume that $n + 1 \geq m$ and let $\beta : A \to \{1, 2 \ldots\}$ satisfy $\sum_{x \in A} \beta(x) = n + 1$. Let $R^N \in L^N$. A *feasible elimination procedure* (f.e.p.) for R^N is a sequence $(x_1, C_1; \ldots; x_{m-1}, C_{m-1}; x_m)$ such that

1) $A = \{x_1, \ldots, x_m\}$,
2) $C_1, \ldots, C_{m-1}$ are pairwise disjoint subsets of N and $|C_j| = \beta(x_j)$ for $j = 1, \ldots, m-1$,
3) For all $j = 1, \ldots, m-1$, $x_kR^ix_j$ for all $k = j+1, \ldots, m$ and $i \in C_j$.

Thus, in a feasible elimination procedure, we can consecutively eliminate bottom alternatives $x_1, x_2, \ldots, x_{m-1}$ of the preference profile; when we eliminate an alternative x_j we also eliminate the preferences of $\beta(x_j)$ voters who have x_j at bottom in the current profile. From the assumptions in the definition it easily follows that there exists always at least one f.e.p. Henceforth in this paper we assume $n + 1 \geq m$. An alternative y is R^N*-maximal* if there exists an f.e.p. $(x_1, C_1; \ldots; x_{m-1}, C_{m-1}; y)$. We denote

$$M(R^N) = \{x \in A : x \text{ is } R^N\text{-maximal}\}.$$

It is not difficult to see that M is an anonymous and Paretian social choice correspondence.[2] Also, M is Maskin monotonic. This follows from Lemma 3.7 below.

[1] Peleg (1978) has been followed by several investigations of the set of exactly and strongly consistent social choice functions: Dutta and Pattanaik (1978), Polishchuk (1978), Ishikawa and Nakamura (1980), Oren (1981), Kim and Roush (1981), Holzman (1986), and Peleg and Peters (2006). Also the books of Peleg (1984), Abdou and Keiding (1991), Danilov and Sotskov (2002), and Peleg and Peters (2010) contain lengthy analytic discussions of consistent voting systems.

[2] Clearly, M depends on β, but this is suppressed from notation if no confusion is likely.

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