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Social groups and social network formation ☆

Bassel Tarbush [a,*], Alexander Teytelboym [b]

[a] Department of Economics and Merton College, University of Oxford, United Kingdom
[b] Institute for New Economic Thinking at the Oxford Martin School, University of Oxford, United Kingdom

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ABSTRACT

We present a dynamic model of social network formation in which a fixed number of agents interact in overlapping social groups. We derive several results on the formation of links in such networks, including results on the degree distribution, on comparative statics relating degree and group size, and on the dynamics of homophily. In particular, we derive comparative statics showing that degree is typically positively related to social group size but negatively related to the size of the overlap across multiple social groups. This is supported by evidence from a Facebook dataset. We also show that homophily over an agent's lifespan in the network can be non-monotonic, reaching a global maximum in some period before eventually decreasing.

1. Introduction

Friendships are an essential part of economic life. Friendships result in peer effects, which impact educational performance (Sacerdote, 2001), health (Kremer and Levy, 2008), group lending (Banerjee et al., 2012), and productivity at work (Falk and Ichino, 2006). The structure of friendships can be described by a social network.[1] How friendships form is key to understanding the properties of social networks and "one of the most important areas of network research [is] developing richer, but still tractable, models of network formation." (Jackson, 2014, p. 17).

The theoretical model presented in this paper is a new dynamic network formation process in which a fixed number of agents interact in overlapping social groups. In every period, every agent interacts with others with a probability that depends on mutual social group sizes and on the sizes of their overlaps (as well as on a set of network-level parameters). When interacting with others in a social group, an agent forms a friendship with another agent chosen at random from among those in the group who are not yet his friends. An example we have in mind is the formation of friendships among college students. A college freshman interacts with students in his class and his dorm. The sizes of these two social groups and the size of their overlap (number of students who are both in his class and in his dorm) determine his chance of becoming friends with students either from his class or his dorm or both.

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* Corresponding author.
E-mail addresses: bassel.tarbush@economics.ox.ac.uk (B. Tarbush), alexander.teytelboym@inet.ox.ac.uk (A. Teytelboym).

[1] The best recent summaries of applications of networks in the social sciences are by Jackson (2008), Goyal (2009), Easley and Kleinberg (2010), and Newman (2010).

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In this paper, we derive several properties of social networks that arise from our network formation process, including results on the degree distribution, on comparative statics relating degree and group size, and on the dynamics of degree and of homophily – the propensity of agents to be friends with others who are similar to themselves.

Our dynamic network formation process has two key features: a fixed number of agents and overlapping social groups. The fixed number of agents in our model drives a number of features of the resulting link formation.[2] For example, we find that agents make friends at a decreasing rate over time because they gradually exhaust the pool of potential friends in each social group. Let us return to our college freshman: after numerous interactions with students in his dorm, he will have become friends with everyone from that social group, and although he continues to interact with students in his dorm, he no longer spends this time making *new* friends, which thus reduces his overall rate of friendship formation. This explanation for the concavity of degree over time contrasts with those given in the literature. In a model with an infinite population of agents who are matched according to probabilities that depend on their types, Currarini et al. (2009) offer one alternative: agents have a decreasing marginal utility of friendships and stop making friends when the marginal cost of a friend exceeds the marginal benefit. The concavity of degree over time is also a feature of many *growing* random network models, in which new agents enter in each period and form links with pre-existing agents, who are chosen according to a specific stochastic process, which may depend on the number of links of the pre-existing agents (Barabási and Albert, 1999; Jackson and Rogers, 2007) or on their characteristics (Bramoullé et al., 2012). In these models, agents make friends at a decreasing rate because they are less likely to receive a link from the incoming agent as the population grows over time. This paradigm is well suited for the analysis of social networks in which the growth of the network is important in capturing features of link formation, whereas focusing on a fixed network size, as we do here, may better capture situations in which there is relatively low volatility in the network growth relative to the rate of link formation.

Our second key feature – overlapping social groups – allows us to derive nuanced comparative statics on the relationship between an agent's degree and the size of the social groups that the agent belongs to. For example, if our freshman is studying for a degree in economics and is resident of a particular dorm, then one might ask what the effect of being in a larger dorm would be, *ceteris paribus*, on his number of friends. The problem is that the ceteris is not paribus in a network (or college) of fixed size. If the freshman's dorm is now larger, then the size of some other social group must have changed for the total number of students to remain unchanged. In other words, one must keep track of what new students joined the dorm. If they were previously in the freshman's economics classes, then their joining the dorm increases the overlap across the freshman's social groups, which, as we show, has a negative impact on his expected number of friends. On the other hand, if non-economists join the dorm, then this positively impacts the freshman's number of friends. Our comparative statics on varying the size of *overlapping* social groups are novel in the literature. Currarini et al. (2009) show that agents belonging to larger groups have higher degrees. However, in their model, there is only one group per agent (e.g. race), so the interaction across social groups cannot be studied. de Marti and Zenou (2011) and Iijima and Kamada (2014) consider strategic network formation models in which the costs and benefits of link formation depend on agents' social groups.[3] de Marti and Zenou (2011) study segregation patterns that arise in stable networks as a function of relative costs and benefits of link formation between and across social groups, whereas Iijima and Kamada (2014) show how properties of stable networks (such as clustering and average path length) depend on "social distance" parameters. However, these papers examine neither dynamics of link formation nor the effect of varying social group sizes and their overlaps.

We also derive results on the dynamics of homophily. Homophily is a commonly observed empirical phenomenon (Kandel, 1978; Shrum et al., 1988; McPherson et al., 2001; Moody, 2001; Mouw and Entwisle, 2006; Mayer and Puller, 2008; Currarini et al., 2009; Wimmer and Lewis, 2010), and most empirical studies of homophily have used surveys of close friendships. For example, Shrum et al. (1988) find that for school children homophily in gender falls over time, but that homophily in race increases over time. In a theoretical paper, Bramoullé et al. (2012) derive a negative relationship between homophily and time and find some empirical support for their prediction in physics citation networks. In contrast to previous work, we show that in our model, homophily is not necessarily monotonic in an agent's degree or in the amount of time that the agent has spent making friends. We provide sufficient conditions on the effective social group sizes for (i) homophily to monotonically decrease over time, and (ii) to increase up to peak and eventually fall over time.

One interpretation of our network formation process – that we use in a running example throughout the paper – is to consider it as a model of friendship formation in online social networks, such as Facebook. "Friending" – recording friendships on online social networking platforms – is different from maintaining real-world friendships. Indeed, different types of friendships – close, distant, romantic or online – generate remarkably dissimilar social networks (Jackson, 2008; Newman, 2010). Most people have few close friends and even fewer lovers. Many platforms, such as Facebook and LinkedIn, provide a record of its users' real-life meetings – an online Rolodex. Typically, after meeting each other, people "send friend requests" in order to record the meeting on Facebook and maintain a "Facebook friendship". For this reason, many Facebook users have more Facebook friends than friends with whom they interact daily.[4] Our model complements growing random

[2] Watts and Strogatz (1998) examine dynamics on a network of fixed size but their linking process does not depend on agent characteristics, as it does here.

[3] These models are in the spirit of Jackson and Wolinsky (1996) and Bala and Goyal (2000), but these earlier contributions did not consider the effect of social groups.

[4] Therefore, rather than reflect close real-world relationships, many Facebook friendships represent "weak ties", which play an important role in economic and social outcomes (Granovetter, 1973, 2005).