Games and Economic Behavior 105 (2017) 9–39

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Balanced ranking mechanisms ☆

Yan Long [a], Debasis Mishra [b,*], Tridib Sharma [c]

[a] NYU, Abu Dhabi, United Arab Emirates
[b] Indian Statistical Institute, Delhi, India
[c] ITAM, Mexico

ARTICLE INFO

Article history:
Received 12 February 2017
Available online 14 July 2017

JEL classification:
D82
D71
D02

Keywords:
Budget-balanced mechanisms
Green–Laffont mechanism
Pareto optimal mechanism

ABSTRACT

In the private values single object auction model, we construct a *satisfactory* mechanism – a symmetric, dominant strategy incentive compatible, and budget-balanced mechanism. The mechanism converges to efficiency at an exponential rate. It allocates the object to the highest valued agent with more than 99% probability provided there are at least 14 agents. It is also ex-post individually rational. We show that our mechanism is optimal in a restricted class of satisfactory *ranking* mechanisms. Since achieving efficiency through a dominant strategy incentive compatible and budget-balanced mechanism is impossible in this model, our results illustrate the limits of this impossibility.

1. Introduction

How should a group of agents allocate a unit of resource among themselves? For instance, consider the problem of allocating a bequest among a group of potential heirs. Many a times, no will exists. Even when a will exists, disputes arise. Designated estate agents are often employed to resolve bequest related problems. A Wall Street Journal article quotes an expert suggesting the following dispute resolution procedure:

> *In family disputes, Ms. Olsavsky says, one option is to have all the items put up for auction. Family members can bid on what they want. The money goes back to the estate to be divided equally (Coombs, 2013).*

There are a number of other examples: a group of firms sharing time slots on a jointly owned supercomputer (Guo et al., 2011); reallocation of the winning good in a bidding cartel (McAfee and McMillan, 1992); a group of municipalities deciding on the location of a stadium (Cramton et al., 1987). A key feature of these problems is that transfers can be used (either as taxes or subsidies) for resource allocation. However, transfers across agents have to balance – for instance, money raised by auctioning a bequest must be redistributed among the heirs.

☆ This paper is a merger of two independent papers: Mishra and Sharma (2016) and Long (2016). We thank the editor and the associate editor for encouraging us to merge the papers. We are grateful to the referees for their comments. The paper has benefited from very useful and detailed suggestions of Rahul Deb, Matt Jackson, Herve Moulin, and Takuro Yamashita. We are also grateful to Dilip Abreu, Attila Ambrus, Claude d'Aspremont, Sushil Bikhchandani, Olivier Bochet, Bhaskar Dutta, Ed Green, Eric Maskin, Motty Perry, Arunava Sen, Yves Sprumont, Ricard Torres, Levent Ulku, Dan Vincent and seminar participants at ITAM, ISI Delhi, University of Glasgow, Canadian Economic Theory Conference, GAMES 2016, Social Choice and Welfare Meeting 2016, and Delhi School of Economics for useful comments.

* Corresponding author.
E-mail addresses: yl79@nyu.edu (Y. Long), dmishra@isid.ac.in, dmishra@gmail.com (D. Mishra), sharma@itam.mx (T. Sharma).

http://dx.doi.org/10.1016/j.geb.2017.07.002
0899-8256/

Table 1
Convergence in our mechanism.

No of agents	Probability to the highest valued agent
9	92.3%
10	95%
11	96.2%
12	98.1%
13	98.9%
14	99.4%
15	99.6%
16	99.8%
17	99.9%

We design mechanisms for such problems with the aim of achieving efficiency. Efficiency requires one to allocate the bequest to the highest valued heir. In the standard private values model, where each agent has a value for the unit of resource/object and transfers are allowed with quasilinear utility, the Vickrey auction satisfies three compelling desiderata of a mechanism: (a) dominant strategy incentive compatibility (DSIC), (b) (allocative) efficiency – allocating the object to the highest valuation agent, and (c) ex-post individual rationality. A well-known criticism of the Vickrey auction is that it is not budget-balanced – it collects revenue from the agents, which distorts ex-post efficiency. Green and Laffont (1979) show that this criticism applies to every DSIC and efficient mechanism: no DSIC and efficient mechanism can be budget-balanced. We look for a second-best solution, where we explore the limits of this impossibility result:

How close to efficiency can we get using a DSIC and budget-balanced mechanism?

We require our solution to satisfy symmetry – agents with identical valuation must get the object with equal probability and pay the same amount. Symmetry is a compelling fairness property – for instance, in the bequest allocation problem, an asymmetric mechanism may either be unacceptable to potential heirs or lead to unpleasant lawsuits later on.

We identify a class of DSIC, budget-balanced, and symmetric mechanisms that we call *ranking mechanisms*. A ranking mechanism is one that uses a *ranking* allocation rule, which is specified (for n agents) by n numbers $(\pi_1, \ldots, \pi_n)$ between 0 and 1 such that they add up to not more than 1 and $\pi_j \geq \pi_{j+1}$ for each j. For every j, the number π_j is the probability with which an agent with the j-th highest value is allocated the object at any generic profile of values. Our main result is a description of the *r-optimal* mechanism – a DSIC, budget-balanced, and symmetric ranking mechanism that beats every such mechanism in terms of the allocation probability to the highest valued agent.

We show that the probability with which the highest valued agent gets the object in our mechanism converges to 1 at an exponential rate. At every profile of values, our r-optimal mechanism allocates the object to the highest valued agent with more than 99% probability, provided there are at least 14 agents. It is also ex-post individually rational. The welfare generated by the r-optimal mechanism converges to efficiency as the number of agents increase. The nature of convergence is shown in Table 1, where we report on the probability with which the highest valued agent gets the object in our mechanism.

The r-optimal mechanism we identify satisfies ex-post individual rationality. Ex-post individual rationality is a desired property of mechanisms ensuring participation.

Ranking mechanisms contain two familiar DSIC, budget-balanced, and symmetric mechanisms: (i) the mechanism that allocates the object to each agent with equal probability without using any transfers and (ii) the *residual claimant* mechanism in Green and Laffont (1979). The residual claimant mechanism is defined by choosing an agent uniformly at random as a residual claimant and conducting a Vickrey auction among the other agents. The revenue generated from the auction is then given to the residual claimant. We refer to this mechanism as the Green–Laffont (GL) mechanism, and note that at profiles of distinct values, it allocates the object to the highest valued agent with probability $1 - 1/n$ and to the second highest valued agent with probability $1/n$.[1] Our r-optimal mechanism coincides with the GL mechanism if the number of agents is no more than 8 but differs from it significantly for more than 8 agents.

Our analysis is prior-free. We use DSIC as our solution concept. As we discuss later in Section 6, Cramton et al. (1987) show that Bayesian incentive compatible, efficient, and budget-balanced mechanisms satisfying a form of individual rationality exists in our model. While the mechanism they propose require information about beliefs of agents (with common prior assumption), our result shows the level of efficiency that can be achieved using DSIC and budget-balanced mechanisms, thus showing the limits of such a prior-free and robust approach in this problem. Recent literature in mechanism design has been investigating such questions in other models (Moulin, 2009; Carroll, 2015).[2]

[1] This mechanism (and its variants) were discussed in the context of public-good provision problem in Green and Laffont (1979). Later, Gary-Bobo and Jaaidane (2000) formally define this mechanism and study its statistical and strategic properties.

[2] There are two recent papers which also provide foundational results of DSIC mechanisms in the private values single object auction environment. Manelli and Vincent (2010) show that in such models, for every Bayesian incentive compatible mechanism, there is an "equivalent" DSIC mechanism – this