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A simple market-like allocation mechanism for public goods

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ABSTRACT

We argue that since allocation mechanisms will not always be in equilibrium, their out-of-equilibrium properties must be taken into account along with their properties in equilibrium. For economies with public goods, we define a simple market-like mechanism in which the strong Nash equilibria yield the Lindahl allocations and prices. The mechanism satisfies critical out-of-equilibrium desiderata that previously-introduced mechanisms fail to satisfy, and always (weakly) yields Pareto improvements, whether in equilibrium or not. The mechanism requires participants to communicate prices and quantities, and turns these into outcomes according to a natural and intuitive outcome function. Our approach first exploits the equivalence, when there are only two participants, between the private-good and public-good allocation problems to obtain a two-person public-good mechanism, and then we generalize the public-good mechanism to an arbitrary number of participants. The results and the intuition behind them are illustrated in the familiar Edgeworth Box and Kölm Triangle diagrams.

There is a substantial literature analyzing the design of institutions, or "mechanisms," for achieving efficient allocations in the presence of public goods. The pioneering paper by Groves and Ledyard (1977) introduced the first public goods mechanism with Pareto efficient Nash equilibria. Subsequently, Hurwicz (1979) and Walker (1981), building on the ideas in Groves & Ledyard, defined mechanisms that attain Lindahl allocations — allocations that are individually rational as well as Pareto efficient. Subsequent theoretical research has focused on developing mechanisms with additional desirable properties, or mechanisms that can be applied to economies with other kinds of externalities.[1]

A number of the mechanisms developed in this theoretical research have been the subject of experimental studies.[2] The experimental results have been mixed at best. The mechanisms have variously failed to converge to equilibrium, or have exhibited slow convergence, or while out of equilibrium have suffered failures of individual rationality, failures of collective feasibility, or severely inefficient outcomes. These results are serious red flags for practical implementation. They suggest that in the case of public goods there remains a gap between implementation in theory and implementation in practice, and that perhaps a different approach might be fruitful.

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[1] For example, Bagnoli and Lipman (1989), de Trenqualye (1989, 1994), Kim (1993), Varian (1994), Peleg (1996), Corchon and Wilkie (1996), Tian (2000), Chen (2002), Healy and Mathevet (2012), and Van Essen (2013, 2015).

[2] For example, see Chen and Plott (1996), Chen and Tang (1998), Chen and Gazzale (2004), Healy (2006), Van Essen (2012), and especially Van Essen et al. (2012).

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The failures when out of equilibrium are especially troubling. Even for mechanisms that have good stability properties, we can't realistically expect to be in equilibrium very often (if ever!). And the failures when out of equilibrium are often unacceptable: outcomes that make some (or all) of the participants far worse off than they would have been had they been simply left alone, or outcomes that are not well-defined, because they are not feasible for some individuals or for the economy as a whole. This suggests that a focus on just the equilibrium properties of mechanisms — asking whether the equilibria are Pareto efficient, or Lindahl allocations, etc. — and even expanding the focus to the mechanisms' stability properties as well, is too narrow. It suggests that we also need to take into account some desiderata for mechanisms' out-of-equilibrium properties.

In this paper we take a very limited, preliminary step in that direction. Our objective is to devise a mechanism for public goods that *always*, whether in equilibrium or not, produces feasible and "acceptable" outcomes and still produces Lindahl allocations as equilibria. Along the way, we introduce a notion of acceptability that, as far as we know, has not appeared before. We're partially successful in attaining this objective: the mechanism we introduce always produces feasible and acceptable outcomes, and does produce Lindahl allocations as Nash equilibria. Communication among the participants is via natural, market-like proposals involving quantities and Lindahl-like prices, which are turned into outcomes according to a transparent and simple outcome function. There are many other, non-Lindahl Nash equilibria as well, however the Lindahl equilibrium is the only strong equilibrium, and the mechanism always yields (weak) Pareto improvements, whether in or out of equilibrium. In the two-person case we also identify the coalition-proof Nash equilibria.

We begin by making two observations. The first is that for the problem of allocating or exchanging purely private goods, simple market institutions such as the double auction have enjoyed remarkable success in the laboratory, beginning with the landmark paper by Smith (1962). The second observation is that when there are only two traders, the public-goods allocation problem and the private-goods allocation problem are essentially identical.

Building on these two observations, we first define a simple market-like mechanism for implementing Walrasian allocations when there are only two consumers and two goods. In the spirit of the market games introduced by Shapley and Shubik (1977), Dubey (1982), and others,[3] the actions the mechanism makes available to the players are natural, economically meaningful price-and-quantity proposals, and the proposals lead to outcomes in a natural and intuitive way. The mechanism's outcomes are always feasible, both for individuals and in the aggregate, whether the mechanism is in equilibrium or not, and the outcomes are always individually rational.

We then show that a straightforward reinterpretation of quantities and prices converts the mechanism into one for allocating a public good. And it's then straightforward to extend the mechanism to an arbitrary number of participants, preserving all the properties of the two-person private-goods and public-good versions of the mechanism.

We make liberal use of the Edgeworth Box to depict the arguments and the intuition for the private-goods exchange mechanism and the Kölm Triangle to provide intuition for the public-goods version of the mechanism.

1. The pure exchange allocation problem

There are two goods and two traders. Trader S wishes to sell good X in exchange for good Y, and Trader B wishes to purchase good X in exchange for good Y. It's convenient to think of Y as money.

The number of units of X the traders exchange will be denoted by q; the price at which the units are exchanged is denoted by p; and we write $m = pq$ for the amount of money exchanged. Thus, B pays $m = pq$ dollars to S in exchange for q units of X. Each trader $i \in \{B, S\}$ has a strictly quasiconcave utility function $u_i(\cdot)$ over trades $(q, m) \in \mathbb{R}^2_+$. We assume that u_S is strictly decreasing in q and strictly increasing in m, and that u_B is strictly increasing in q and strictly decreasing in m. See Fig. 1.

While we define the mechanism and carry out the analysis in terms of trades (q, m) and the associated prices p, these can of course be related to allocations and preferences in the usual way:

Each trader $i \in \{B, S\}$ owns an **endowment bundle** $(\mathring{x}_i, \mathring{y}_i) \in \mathbb{R}^2_+$, with $\mathring{x}_S > 0$ and $\mathring{y}_B > 0$, and the mechanism's outcome (q, m) yields the **allocation** $((x_B, y_B), (x_S, y_S))$ defined by

$$x_B = \mathring{x}_B + q, \quad y_B = \mathring{y}_B - m, \quad x_S = \mathring{x}_S - q, \quad y_S = \mathring{y}_S + m. \tag{1}$$

We assume that each trader has a strictly quasiconcave utility function U_i over bundles $(x_i, y_i) \in \mathbb{R}^2_+$, from which the functions u_i above are defined in the obvious way (see Fig. 1):

$$u_B(q, m) = U_B(\mathring{x}_B + q, \mathring{y}_B - m) \quad \text{and} \quad u_S(q, m) = U_S(\mathring{x}_S - q, \mathring{y}_S + m).$$

For any price $p > 0$, let $\widehat{q}_i(p)$ denote Trader i's utility-maximizing quantity q_i — i.e., the trade $(\widehat{q}_i, p\widehat{q}_i)$ maximizes $u_i(q_i, pq_i)$ for the given price p. Thus, $\widehat{q}_B(\cdot)$ is Trader B's demand function and $\widehat{q}_S(\cdot)$ is Trader S's supply function. Note that we use $\widehat{q}_i$ to denote both the function $\widehat{q}_i(\cdot)$ and also the quantity $\widehat{q}_i(p)$, when it's clear what the relevant price p is.

We restrict our attention to allocation problems in which there is a unique Walrasian allocation, which we assume is interior: the **Walrasian outcome**, denoted (q^W, p^W), is the unique pair (q, p) that satisfies $q = \widehat{q}_B(p) = \widehat{q}_S(p)$, and we assume that $0 < q^W < \mathring{x}_S$ and $0 < p^W q^W < \mathring{y}_B$.

[3] For example, Wilson (1978), Schmeidler (1980), and Binmore (1987).

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