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Optimal allocation of an indivisible good ☆

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A B S T R A C T

In this paper, we consider the problem of allocating an indivisible good efficiently between two agents with monetary transfers. We focus on allocation mechanisms that are dominant-strategy incentive compatible when agents' types are private information. Inefficiency of an allocation mechanism may come from two sources: misallocation of the indivisible good and an imbalanced budget. Unfortunately, as Green and Laffont (1979) demonstrate, no allocation mechanism can always overcome both kinds of inefficiency. We identify allocation mechanisms that maximize the expected total utilities of agents, and characterize optimal mechanisms for a large class of agents' type distributions. For strongly regular type distributions, we show that the optimal mechanisms must be budget-balanced: they are either fixed-price mechanisms or option mechanisms. The result may not hold for other type distributions. For certain type distributions, we show that optimal mechanisms are hybrids of Vickrey–Clarke–Groves mechanisms and budget-balanced mechanisms.

1. Introduction

In this paper, we consider the problem of allocating an indivisible good efficiently between two agents when agents' valuations of the good are private information. A typical example of such a problem is the bilateral bargaining problem, in which a seller and a buyer negotiate over whether and how to trade a particular good. Our focus is on dominant-strategy incentive compatible mechanisms. The research interest in this problem is derived from a fundamental dilemma of Green and Laffont (1979): When agents' valuations of the good are private information, it is impossible to always assign the good to the agent with the higher valuation without incurring any cost.

There are several methods that are commonly used in practice, including lotteries, seniority rankings, auctions. These methods either sometimes assign the good to the agent with the lower valuation or sometimes incur negative cash outflows from agents.

For scholars, two particular classes of methods have received more attention. The first class consists of all Vickrey–Clarke–Groves (VCG, henceforth) mechanisms (Vickrey, 1961; Clarke, 1971; Groves, 1973) that extend the conventional

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English auction scheme. The second class consists of all fixed-price mechanisms (Hagerty and Rogerson, 1987), in which the good is assigned to one agent (the seller) unless both agents are willing to trade the good at a predetermined price. VCG mechanisms always assign the good to the agent with the highest valuation, but they may incur outflow of money from agents (money burning). Fixed-price mechanisms do exactly the opposite.

Although extensive research has been conducted on VCG mechanisms and fixed-price mechanisms separately, they have never been scored against each other in any formal model, let alone in a model that allows for more-general mechanisms. Note that VCG and fixed-price mechanisms share two common features. First, they are dominant-strategy incentive compatible—i.e., it is always a dominant strategy for agents to reveal their types truthfully. Second, they are no-deficit—i.e., they have no need for money injection from outside to facilitate the agents. In this paper, we shall study all mechanisms that are dominant-strategy incentive compatible (DSIC) and no-deficit (ND). Our goal is to identify the optimal mechanisms among them.

To evaluate DSIC and ND mechanisms we assume a known Bayesian prior over the private types of the agents and look for mechanisms that perform well in expectation over types from this prior. Note that a corollary of the work by Green and Laffont (1979) is that there exists no mechanism that is always more efficient than others in every realization of agents' types. Our Bayesian objective is a standard one for mechanism design in environments where no mechanism is pointwise optimal.[1]

In Theorem 1, we present a characterization of optimal mechanisms when agents' type distributions are strongly regular.[2] An optimal mechanism is either a fixed-price mechanism or an option mechanism, depending on agents' type distributions. Hence, any optimal mechanism must be budget-balanced. Both fixed-price and option mechanisms are optimal if agents are identical ex ante. In an option mechanism, one agent is the temporary holder of the good, and the other agent is the recipient of a call option that allows him to purchase the good from the first agent at a predetermined price. The good changes hands whenever the option recipient wants to exercise his option. In comparison, under the fixed-price mechanism, the good changes hands only when both agents agree to the trade at a predetermined price. When agents' types are not strongly regular, the conclusions in Theorem 1 no longer hold. We study several such cases in Theorems 2 and 3 when agents are symmetric *ex ante*, obtaining characterizations of optimal mechanisms. Optimal mechanisms in these more general cases are not always budget-balanced, as they might be hybrids of VCG and budget-balanced mechanisms: An optimal mechanism may sometimes assign the good efficiently and sometimes impose budget-balance depending on the type profile.

We believe that our results make a significant contribution to the literature on mechanism design, as there are very few examples of closed-form optimal dominant-strategy incentive compatible mechanisms. Moreover, Theorem 1 highlights the importance of budget-balancedness for optimality with strongly regular type distributions. On the other hand, Theorems 2 and 3 demonstrate that the optimal mechanisms need not be either VCG mechanisms or budget-balanced mechanisms in other cases. They complement discoveries found by Miller (2012), Drexl and Kleiner (2015), and Schwartz and Wen (2012) through examples that either budget-balanced or VCG mechanisms can be outperformed by other mechanisms on average for different type distributions.

RELATED WORK. This paper considers dominant strategy incentive compatible and ex post no-deficit mechanisms to allocate a good between two agents to maximize the expected agents' utilities when the agents' types are drawn from a known distribution. Guo and Conitzer (2010) consider a generalization of our problem with multiple goods and multiple agents and look for VCG mechanisms (which always choose the surplus maximizing allocation) that minimize the expectation of the outflow of money. This outflow of money can be reduced by redistributing the VCG payments among the agents (where the money not redistributed is burnt). Schwartz and Wen (2012) provide an example of a bilateral trade model in which the mechanism with money burning outperforms budget-balanced mechanisms for certain distributions. Miller (2012) shows that VCG mechanisms can never be optimal for a general class of agents' type distributions. Finally, in a contemporaneous paper, Drexl and Kleiner (2015) consider a variant of our problem, in which an additional ex post individual rationality (IR) condition is also imposed on mechanisms. Within this smaller set of mechanisms, they show that the optimal mechanisms are budget-balanced, a result similar to our Theorem 1. The advantage of their work is that their result is valid for all regular distributions,[3] a more general class of distributions than ours. Nevertheless, when the IR condition is dropped, the optimal mechanisms are not necessarily budget-balanced for regular distributions as our Theorems 2 and 3 demonstrate. One must assume strong regularity in order to show that optimality implies budget-balancedness.

There is a line of research that considers a similar question but relaxes the DSIC requirement to Bayesian incentive compatibility. With this relaxation the mechanism of D'Aspremont and Gérard-Varet (1979) obtains the first-best welfare and, consequently, the no-deficit condition imposes no loss. There are two reasons to consider our mechanisms over these

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1 Previous works have considered the same setting but relaxing DSIC to Bayesian incentive compatibility or strengthening the Bayesian optimization criteria to a pointwise objective (but relaxing the optimality criteria to one of approximation). A comparison of these works to ours will be given in detail in the related work section.

2 Our notion of strongly regular distribution requires that both the hazard rate and the reversed hazard rate are monotone. See Section 2 for details.

3 The hazard rates of the type distributions are monotone.