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On the operation of multiple matching markets ☆

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ABSTRACT

We provide a new perspective on how to operate matching markets when there are many types of markets. Our finding is that the market-wise adaptation of *strategy-proof* and *non-wasteful* rules yields a strategy-proof rule with the following efficiency property: no strategy-proof rule Pareto-dominates the rule. Such rules are abundant as they include the market-wise use of the well-known priority-based rules of the top trading cycles and the deferred acceptance. This result gives theoretical support to the independent operation of markets observed in real-life markets as well as our practice in Market Design that separately treats each market for its design.

1. Introduction

We have recently witnessed economists' discovery and design of real-life matching markets such as school choice (Abdulkadiroğlu and Sönmez, 2003) and organ exchanges (Roth et al., 2004; Ergin et al., 2014). In such a market one type of indivisible good, or object for short, is allocated to agents who demand one unit when monetary transfers are not allowed (Shapley and Scarf, 1974; Hylland and Zeckhauser, 1979).

We often deal with multiple types of objects at the same time — what we call a *multiple-type (matching) market*. People simultaneously participate in multiple markets, each of which deals with a different type of commodity. Examples are abundant: Many families live in public housing and have children who go to school — one type of object is public housing and another is a seat in a school[2]; Teaching courses and administrative tasks are assigned to faculty members in universities

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[2] The percentage of the population living in public housing is about 10 to 35 in many countries. For example, Whitehead and Scanlon (2007) report that the percentages in European countries around 2000 are 35 in the Netherlands, 25 in Austria, 21 in Denmark, 20 in Sweden, 18 in England, 17

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Table 1
An example of inefficiency.

Agent 1			Agent 2		
R_1	R_1^1	R_1^2	R_2	R_2^1	R_2^2
(a,c)	$\underline{a}$	c	(a,c)	a	$\underline{c}$
(b,c)	b	$\underline{d}$	(a,d)	$\underline{b}$	d
$\underline{(a,d)}$			$\underline{(b,c)}$		
(b,d)			(b,d)		
$\vdots$			$\vdots$		

Note: Agent i's preference for bundles is denoted by R_i while that for type-t objects is by R_i^t. For example, agent 1 prefers bundle (a,c) to (b,c) to (a,d), ..., and she prefers a to b for type 1 while she does c to d.

where one type of object is courses and another is administrative tasks; About 80% of pancreas patients in Japan also need a kidney transplant due to chronic kidney disease where one type of object is the pancreas and another the kidney.[3] Furthermore, a dynamic matching market is a special class of multiple-type markets where people dynamically participate in one market multiple times (Kurino, 2014). This is because the objects consumed in two distinct dates are considered to be different object types. For example, college students take part in the annual assignment of dormitory rooms; Families with several children participate in a school choice program or a daycare assignment program at different times when each of their children enters the school (Dur, 2012; Kennes et al., 2014); Teachers in public schools move to different schools through annual assignment procedures (Bloch and Cantala, 2013).

In this paper, we focus on a multiple-type market. Each agent has a separable preference, where a preference over bundles is *separable if* there is a list of preferences over each single-type market such that if two bundles x and y are different in only t-th type, then the evaluation between x and y coincides with the evaluation of t-th types x^t and y^t according to the preference over the t-th market objects.[4]

One important observation is that in most cases, if not all, assignment procedures for different types of objects are operated by different government authorities, and each procedure is independent from the other.[5] Corresponding to this practice in real-life markets, the Market Design literature treats each market independently and then considers the design of *strategy-proof* and *Pareto efficient* (or at least *non-wasteful*) rules for only one type of market.[6] However, such independent operation of different markets leads to inefficiency when we take all types of markets into account.

To illustrate this point, suppose that there are two agents and two types of objects. We have two objects a, b for type $t=1$ market, and two objects c, d for type $t=2$ market. Preferences of agents 1 and 2 are given by Table 1. Now, let us consider the underlined allocation in this table where agent 1 receives (a,d) and agent 2 gets (b,c). In other words, for type $t=1$ the allocation prescribes $(1,2;a,b)$, while for type $t=2$ it does $(1,2;d,c)$. Note that within each type market the allocations $(1,2;a,b)$ and $(1,2;d,c)$ are Pareto efficient. However, by swapping their bundles both agents are made better off. This means that the original allocation is not Pareto efficient, although its allocation for each type is Pareto efficient. One lesson from this example is that an allocation procedure that uses a Pareto efficient rule for each type might result in an inefficient allocation.[7]

One of the most serious difficulties pertaining to the multiple-type market is that the combination of *strategy-proofness* and *Pareto efficiency* essentially results in a serial dictatorship rule in which each agent chooses her assignment one by one according to an exogenously fixed priority order (Monte and Tumennasan, 2015).[8] However, such a rule is against the independent operation of markets in real life and might be extremely unfair.[9]

in France, 8 in Ireland, 6 in Germany, and 4 in Hungary. Moreover, the percentages are 30 in Hong Kong and 9.9 in Japan. (The webpages are http://www.housingauthority.gov.hk/en/public-housing/index.html and http://www.stat.go.jp/data/jyutaku/2008/10_3.htm. The webs were accessed on December 18, 2013.) This suggests that about the same percentage of families with children would participate in two types of markets — public housing assignment and school choice programs.

[3] For example see the Handbook of The Japan Society for Transplantation (2013).

[4] A model with single-type objects is a special case of ours. Although our main focus is a multiple-type model, all of our results remain true in single-type models.

[5] For example, in the City of Boston, the school choice assignment is operated by Boston Public Schools, while the public housing assignment is controlled by the Boston Housing Authority.

[6] Under a *non-wasteful* allocation (Balinski and Sönmez, 1999), if some object is preferred to the assigned one for an agent, it is fully assigned up to its quota. Thus, *non-wastefulness* is an efficiency axiom weaker than *Pareto efficiency*. Note that *non-wastefulness* is defined only for single-type markets. The lack of a natural second-best efficiency notion in a multiple-type market is another motivation of the current paper.

[7] More specifically, this is a Pareto inefficient allocation although it is reached by the market-wise serial dictatorship rule which is strategy-proof and Pareto efficient in each type market. Agent 1 is the dictator of the first market while agent 2 is the dictator of the second market.

[8] Rigorously speaking, Monte and Tumennasan's (2015) model is not a special case of ours due to the difference in preference domains (see Assumption 2 and footnote 19). They prove that *if* a rule is *strategy-proof*, *Pareto efficient* and *non-bossy*, then it is a sequential dictatorship rule that is a variant of the serial dictatorship rule.

[9] Consider the environment with homogeneous preferences of agents and unit quotas of objects. In the "full" serial dictatorship (SD) rule, the highest-priority agent receives her favorite objects from all types, and is envied by all agents in each type. However, we can mitigate the unfairness with a