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Note

Generalized coarse matching

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This paper analyzes the problem of matching two heterogeneous populations, such as men and women. If the payoff from a match exhibits complementarities, it is well known that, absent any friction, positive assortative matching is optimal. Coarse matching refers to a situation in which the populations are sorted into a finite number of classes and then randomly matched within these classes. We derive upper bounds on the fraction of the total efficiency loss of n-class coarse matching, which is proportional to $1/n^2$. Our result substantially enlarges the scope of matching problems in which the performance of coarse matching can be assessed.

1. Introduction

Consider a canonical matching problem: there are two heterogeneous populations of equal size. Agents within each population have the same preferences regarding the other population. For instance, one population may consist of men and a second of women. Within each population, agents differ by ability, beauty, education, etc., and each prefers agents from the other population who have higher ability, beauty, education, etc. If two agents (e.g., a man and woman) match, their payoffs depend on the characteristics of both partners.

It is well known that if a payoff function exhibits complementarities, it is optimal to match the two populations in a positive assortative fashion to maximize total match payoffs (e.g., Becker, 1973). That is, men with the best (worst) characteristics are matched with the women with the best (worst) characteristics. This method of matching is clearly better than randomly assigning men to women.

This paper focuses on an intermediate method of pairing two populations—namely, *coarse matching*. It proceeds by partitioning the population into n classes and then randomly matching individuals within each class. We ask: how much of total surplus is captured by coarse matching (henceforth, CM) when compared with the total surplus generated by positive assortative matching (henceforth, PAM)?[2] Before presenting the result, let us explain the importance of the question.

Note that PAM requires a matchmaker (planner) who knows the actual type of each member of each population. When the populations are large, this is a strong informational requirement. In practice, there is a cost of acquiring information on agents' types, but this cost is not modeled in typical analyses. We would think that the cost of acquiring perfect information

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[2] In this paper, by PAM, we mean perfect sorting, although CM also involves positive assortative matching in a very coarse sense.

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is significantly greater than the cost of acquiring coarse information. Given these costs, should the social planner implement PAM or settle for CM? The answer depends on whether the efficiency gain of using PAM over CM is small or large. If it is small, it may not justify the cost of acquiring this additional information.[3]

This question was first addressed by McAfee (2002). He considers coarse matching, with each population partitioned into *exactly* two classes. He shows that when the match payoff function takes a multiplicative form and distributions of attributes satisfy certain hazard rate conditions, CM can lose no more than half of the total surplus generated by PAM.

We tackle this problem and investigate the performance of coarse matching (i.e., the fraction of the efficiency loss). The main result shows that for each n, we can construct an n-class CM with an upper bound on its efficiency loss. In particular, this upper bound is an expression that is proportional to $1/n^2$. We also show by example that this upper bound is tight. The coefficient of the upper bound has two parts: the first part measures the efficiency loss due to the variation in the complementarity (cross derivative) of the match payoff; and the second part captures the efficiency loss due to agents' type distributions. Our result also separates these two effects.

Our main result generalizes McAfee's result along several dimensions by allowing: 1) arbitrary match payoff functions that are complementary in agents' characteristics; 2) almost any distributions of types; and 3) an arbitrary fixed number of classes. Note that most results in the literature are derived by assuming that match payoff is multiplicative in agents' types. Our paper, hence, substantially enlarges the class of matching problems to include almost any in which the performance of CM can be assessed.

While Chebyshev's sum inequality plays the central role in McAfee (2002), one of our technical contributions is the novel use of the so-called "Grüss's inequality" to establish our result, which could be useful for future studies.

RELATED LITERATURE. Wilson (1989) considers a mathematically equivalent model and shows that the efficiency loss of using n-class CM converges to zero at a rate of $O\left(1/n^2\right)$. Nevertheless, the convergence rate is not quite informative for understanding the performance of pricing schedules with a given number of priority groups, especially when this number is not large. McAfee (2002) rephrases that model as a two-sided coarse matching model. He proves that under a restriction to a certain class of distributions, 2-class CM has no more than half of the efficiency loss. Hoppe et al. (2011) further refine McAfee's (2002) result with a tighter bound and derive new bounds for other classes of distributions. By applying these results to a monopolistic pricing problem with private information, they analyze the performance of coarse matching in terms of output, revenue and agents' welfare. However, they still restrict their analysis to 2-class CM with multiplicative payoff functions.

Our paper proceeds as follows: In Section 2, we introduce the formal model, notations and assumptions. In Section 3, we present our main finding. Section 4 concludes. The Appendix contains the details of proofs.

2. The model

A two-sided market consists of two equal-sized populations of agents. For clarity, we refer to the agents of each population as men and women, each of whom is characterized by type. Denote the types as x and y, respectively. Agents' types are distributed over $[0, 1]$ according to distribution functions $F(x)$ and $G(y)$.[4] Throughout the paper, we assume that corresponding density functions $f(x)$ and $g(y)$ are continuous and strictly positive over $(0, 1)$. We denote the mean μ_x and μ_y, respectively.

Each agent is assumed to be matched with one agent from the other side of the market—that is, one man may marry only one woman. For matched agents with types x and y, the match payoff is $m(x, y)$, which is assumed to be strictly complementary in x and y and smooth enough—i.e., $m_{12} = \dfrac{\partial^2}{\partial x \partial y} m(x, y) > 0$. We assume that m_{12} is bounded by $\underline{m}$, $\overline{m}$ from below and above.[5]

We are interested in the overall match payoff of all agents. It is well known in the literature (e.g., Becker, 1973) that if a match payoff function exhibits complementarity in agents' types, the optimal allocation involves PAM. In other words, the highest-type man matches the highest-type woman, and the second-highest-type man matches the second-highest-type woman, and so on. Denote the agent's percentile as i indexed by the rank of his/her type in each population. With a slight abuse of notation, let $x(i)$ and $y(i)$ be agents' types given their percentiles, respectively, with $x(i) = F^{-1}(i)$ and $y(i) = G^{-1}(i)$. Apparently, i is uniformly distributed over $[0, 1]$.

3 This is not the only justification for CM. For example, McAfee (2002) argues, "The use of a continuum of priorities is not feasible in many circumstances—using many priorities makes the scheme unwieldy to administer and opaque to consumers. Moreover, if the priority prices are determined by bidding, as is natural, the auction process will be complex and expensive to operate when there are many service classes." Hoppe et al. (2011) suggest that "[t]hese costs may take the form of: communication, complexity (or menu), and evaluation costs for the intermediary (who needs more detailed information about the environment in order to implement a fine scheme), and for the agents (who need precise information about their own and others' attributes in order to optimally respond to a fine scheme), or higher production costs for firms offering different qualities."

4 Our results hold for any positive type space that is an interval with finite length. Without loss, we just assume $[0, 1]$ for simplicity.

5 The strict complementarity is assumed for technical convenience to estimate the match surplus of PAM over random matching, $U_\infty - U_1$ defined later in the paper, which is then used to measure efficiency loss of CM in the main theorem. Note that $U_\infty - U_1$ is independent of the formation of any CM. It can be directly calculated as a parameter of the matching problem without this assumption.