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Note

When does restricting your opponent's freedom hurt you?

Harry Di Pei [1]

Department of Economics, Massachusetts Institute of Technology, United States

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ABSTRACT

I examine the payoff consequences for a player when she removes a subset of her opponent's actions before playing a two-player complete information normal form game. When she faces a constraint on the maximal number of actions she can remove, she can be strictly better off by *not* removing any actions. I present such an example. I also establish sufficient conditions under which removing opponent's actions cannot hurt. As a corollary, I also characterize a necessary condition for a player's optimal Nash Equilibrium in games with generic payoffs when her opponent has strictly more actions than she does.

1. Introduction

Consider a complete information normal form game played by two players. Suppose before the game starts, one of the players, say player 1 (she), can choose to remove some of player 2's (he) actions from his action set. After that, players proceed to play the '*restricted game*'. Suppose player 1 only has the capability of removing k or fewer actions, then when does she prefer to restrict player 2's freedom in a non-trivial way, i.e. removing a non-empty subset of his actions, before the game starts?

This game theoretic question captures a number of economically interesting applications in political economy and organizational economics. For example, consider the two players being two political groups. One of the groups is in power and can restrict the other group's freedom by forbidding certain actions. However, such power is usually limited either due to the cost of policing and enforcement, or due to other exogenous restrictions (for example, the constitution). In my model, this '*limited amount of power*' is captured by the constraint that player 1 can remove at most k of player 2's actions. A larger k means that player 1 is more powerful. Similar problems can also arise within firms and organizations, in which superiors can impose rules on their subordinates, but the cost of monitoring as well as other concerns require her to leave at least some discretion to the latter.

Contrary to the conventional wisdom that restricting the freedom of an opponent should always be beneficial, I start with a counterexample, in which player 1 is *strictly* worse off by deleting any one of her opponent's actions, even when she can choose it optimally and can choose which equilibrium to coordinate on afterwards. The message from this example is clear: when player 1's ability to restrict her opponent's freedom is limited (i.e. she cannot reduce her opponent's action set to a singleton), then there exist circumstances in which she finds it *strictly* optimal to leave full discretion to her opponent.

E-mail address: harrydp@mit.edu.

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Next, I characterize sufficient conditions under which player 1 is weakly better off by removing at least one action from her opponent's action set. Aside from the trivial case in which her opponent has only $k+1$ actions, I show that player 1 is weakly better off when her opponent has strictly more actions than she does. The proof uses the Carathéodory Theorem to construct an equilibrium in the restricted game, which gives player 1 a payoff that is weakly greater than her highest equilibrium payoff in the original game.

As a byproduct of the proof, I also show that in 2-player games with generic payoffs, if player 2 has strictly more actions than player 1, then in every equilibrium that is optimal for player 1, player 2 must be playing a (possibly mixed) strategy that has at most n actions on its support, where n is the number of actions player 1 has.

Related literature: The question I asked is related to the literature on commitment games, *à la*, Renou (2009), Bade et al. (2009), in which there is an *ex ante stage*, during which every player commits to remove a subset of his actions.[2] Instead of examining the payoff consequences of commitment, i.e. a player restricting her own freedom, I examine the payoff consequences of restricting her opponent's freedom.

The underlying message of my paper is related to that in Bernheim and Whinston (1998), who show that if contracts must be incomplete due to non-verifiability, then it is often optimal for players to write contracts that are even more incomplete, i.e. leaving other verifiable aspects unspecified. However, the ways in which we model limiting freedom are very different. In their model, there is a partition for each player's action set and the court cannot distinguish actions within the same partition element. Therefore, under every contract, every player's allowable action set must be measurable with respect to that partition. In my model, all actions are verifiable but there is an upper bound on the number of actions that can be removed. This difference in modeling also leads to different results. In their model, signing an incomplete contract and leaving extra discretion to players is never optimal in static simultaneous move games, which is not true in my model.

2. Model setup

The original game: Consider a complete information normal form game: $\mathcal{G}=(I,A,U)$, where:

- The set of players is $I=\{1,2\}$.
- Player $i\in I$ has a finite action set A_i, with typical element a_i. An action profile is denoted by $a\equiv(a_1,a_2)\in A\equiv A_1\times A_2$.
- $U=(U_1,U_2)$, where $U_i:A\to\mathbb{R}$ maps action profiles to player i's payoff.

Let $\Delta(\cdot)$ be the set of probability measures on a finite set. The mixed extension of this game is defined naturally, with $\alpha_i\in\Delta(A_i)$ being player i's mixed action. For $i\in I$, let

$$U_i(\alpha_1,\alpha_2)\equiv\sum_{a_1\in A_1}\sum_{a_2\in A_2}\alpha_1(a_1)\alpha_2(a_2)U_i(a_1,a_2)$$

be player i's expected payoff from mixed strategy profile (α_1,α_2). Let the cardinalities of A_1 and A_2 be n and m, with:

$$A_1\equiv\{a_1^1,a_1^2,...,a_1^n\},\quad A_2\equiv\{a_2^1,a_2^2,...,a_2^m\}.$$

I focus on the case in which $m\geq 2$.

A (possibly mixed) Nash Equilibrium (NE), (α_1^*,α_2^*), is defined as in Fudenberg and Tirole (1991, hereafter, FT), with $NE(\mathcal{G})$ the set of NEs in game $\mathcal{G}$. FT shows that the set of NEs is closed,[3] and players' payoffs are continuous in (α_1,α_2). As a result, player 1's highest NE payoff exists, which is defined as:

$$U_{1,max}(\mathcal{G})\equiv\max_{(\alpha_1,\alpha_2)\in NE(\mathcal{G})}U_1(\alpha_1,\alpha_2).$$

Restricting freedom: Suppose before the game starts, player 1 (she) can forbid player 2 (he) from playing at most $k\in\mathbb{N}$ of his actions, where $1\leq k\leq m-1$. I view k as a parameter that measures the ability of player 1 to restrict her opponent's freedom.

Let $\tilde{A}_2\subset A_2$ be the set of remaining actions, with $m-k\leq\#\tilde{A}_2\leq m-1$, where '#' denotes the cardinality of a set. After $\tilde{A}_2$ is chosen, players play a '*restricted game*':

$$\mathcal{G}^{\tilde{A}_2}\equiv\left(I,(A_1,\tilde{A}_2),U^{\tilde{A}_2}\right),$$

[2] Romano and Yildirim (2005) studies dynamic games, in which players can only increase their actions over time. In this spirit, playing a higher action in the early stages commits a player to player higher actions in the future.

[3] This is implied by FT's claim in page 30 that players' best reply correspondences have closed graph property, which is a Lemma towards proving the existence of NE.