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Note

Markovian equilibria in dynamic spatial legislative bargaining: Existence with three players ☆

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ABSTRACT

The paper proves, by construction, the existence of Markovian equilibria in a dynamic spatial legislative bargaining model. Three players bargain over one-dimensional policies in an infinite horizon. In each period, a sequential protocol of proposal-making and voting, with random proposer recognitions and a simple majority, produces a policy that becomes the next period's status-quo. An equilibrium exists for any profile of proposer recognition probabilities, any profile of players' ideal policies, and any discount factor. In equilibrium, policies converge to the median's ideal policy, players moderate and propose policies close to the median's ideal in an attempt to constraint future proposers, but the tendency to moderate is a strategic substitute as the opponent of a moderating player does not moderate.

1. Introduction

Legislative policy-making often concerns policies that are continuing in nature, evolve and change over time. Any change to a continuing policy is negotiated under the shadow of the extant legislation, leads to a revision of the extant legislation, and results in a shift of the status-quo. Dynamic legislative bargaining models reflect these features. The models embed a sequential protocol of proposal-making and voting from static non-cooperative legislative bargaining models as a stage game in an infinite horizon dynamic strategic situation. Two consecutive rounds of negotiations are linked; the decision from the former becomes the status-quo for the latter.

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Starting with Baron (1996), the dynamic legislative bargaining literature has been steadily growing. For a *spatial* setting, bargaining over policies, Baron (1996) develops partial equilibrium characterization and provides intuition for the strategic forces at play. For a *distributive* setting, bargaining over the allocation of benefits, Kalandrakis (2004) is the first to characterize a Markov equilibrium. In the absence of applicable existence theorems for Markovian equilibria, his characterization constitutes an existence proof. Due to the lack of similar characterization for the spatial model, and in the continuing absence of applicable existence theorems, the existence and properties of Markov equilibria in the dynamic spatial model remain unknown.

In this paper we prove, using constructive arguments, the existence of Stationary Markov Perfect equilibrium (SMPE) in a dynamic spatial legislative bargaining model. Three legislators repeatedly set policy in a one-dimensional policy space. Their preferences are quadratic around policy ideals, around *bliss points*. In each of an infinite number of periods, one legislator is randomly recognized to propose a policy. The legislature decides either to adopt the policy proposal or to maintain the status-quo policy under a simple majority vote. The winning policy determines the legislators' utility for the period and becomes the status-quo for the next period.

The equilibrium construction relies on two classes of (pure) stationary Markov proposal strategies. A proposal strategy maps the status-quo into a policy proposal. A proposal strategy in the first class depends on a single parameter, the policy a player proposes when the status-quo gives her ample bargaining power. In the static setting this parameter would be the player's bliss point. In the dynamic setting we call this parameter the *strategic bliss point*, the policy maximizing, in equilibrium, the dynamic utility of a player. Because a proposal strategy of a player is fully determined by her strategic bliss point, a profile of strategic bliss points fully determines a profile of proposal strategies, generates the dynamic utility of each player and induces a different profile of dynamic utility maximizers, a different profile of strategic bliss points. The SMPE construction using the proposal strategies in the first class can be seen as finding a fixed-point of this operation.[1]

An SMPE cannot be always constructed using the proposal strategies in the first class because those can be optimal only if the dynamic utilities are single-peaked. The proposal strategies in the second class can be optimal even for non-single-peaked dynamic utilities and allow us to prove the general SMPE existence result. That it is possible to construct an SMPE using the proposal strategies from the two classes is the main insight of the paper.

Moderation and its *strategic substitute* nature are at the core of our equilibrium construction. A player moderates when she proposes her strategic bliss point, a more moderate policy—closer to the median—than her (static) bliss point. Moderation is driven by strategic considerations. The proposing player anticipates her proposal's impact on the future policies. A moderate proposal constrains future proposers, most importantly the current proposer's opponents, to propose moderate policies as well. Moderation is a strategic substitute; when a player's opponents *do* moderate, they are constraining themselves and the player has no incentive to moderate; when a player's opponents *do not* moderate, the player has an incentive to do so.

The equilibrium extent of moderation is a result of two opposing forces. The first force is standard and pushes proposed policies towards the players' ideal policies. The second force is strategic and pushes proposed policies towards the median's ideal policy. The second force gains prominence and the equilibrium extent of moderation increases with the patience of the players and with the probability of recognition of their opponents.

Our work is the most closely related to the dynamic spatial legislative bargaining literature. Baron (1996) is the first to study policy determination with an endogenous status-quo. We study a three-player version of Baron's (1996) model.[2] The partial equilibrium characterization developed in Baron (1996) uncovers moderation as an equilibrium feature. By providing complete equilibrium construction, we uncover not only moderation, but also its strategic substitute nature.[3] Kalandrakis (forthcoming) considers a model identical to ours but assumes equidistant players' bliss points and equal recognition probabilities.[4] Forand (2014) explores an electoral competition model between two parties and a median voter. His key assumption, incumbent policy commitment, creates a link between periods and makes his model closely related to the model in Baron (1996), with an additional restriction that only two players possess agenda setting power and alternate in the proposer role.[5] In a companion paper (Zapal, 2015), we study extension of the model considered here to a multi-player environment. The equilibrium existence result we prove there is limited to a certain class of games and utilizes only the first class of proposal strategies discussed above. The insight that the proposal strategies in the second class can be used to complete general existence result is specific to this paper.

General characterization and existence results for Stationary Markov Perfect equilibria in dynamic legislative bargaining games are scarce. Kalandrakis is the first to characterize an SMPE in a dynamic distributive legislative bargaining model with

[1] The proposal strategies in the first class closely resemble those of Romer and Rosenthal (1979). A player proposes her (strategic) bliss point for any status-quo for which it is accepted, and otherwise proposes the policy closest to it, from among those that are acceptable.

[2] We do not restrict the policy space to $\mathbb{R}_+$, which is of little consequence, and we use quadratic stage utilities, which is necessary and cannot be dispensed with. See discussion following Theorem 1. Baron (1996) does not assume quadratic utilities, but assumes that the dynamic median voter theorem applies. Hence, assuming quadratic utilities does not decrease the generality of the model as compared with Baron (1996).

[3] Baron (1996) includes informal discussion of an example of full equilibrium characterization for a five player game (his Table 1) with symmetric extent of moderation by all non-median players. For three players, Kalandrakis (forthcoming, Proposition 1) shows that symmetric equilibria of the type characterized by Baron (1996) cannot exist. Our Proposition 2 has the same implication and Proposition A4 in Appendix shows that all non-median players moderating is incompatible with a large class of equilibria.

[4] Footnote 13 explores the relationship between his mixed strategy and our pure strategy equilibrium constructions.

[5] A working paper version of Forand (2014) (Forand, 2010) draws the analogy between his electoral model and the legislative bargaining model of Baron (1996). Footnote 14 explores the relationship between his and our equilibrium constructions.