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Blackwell's informativeness ranking with uncertainty-averse preferences ☆

Jian Li [a,*], Junjie Zhou [b]

[a] CIREQ and Department of Economics, McGill University, 855 Sherbrooke Street West, Montreal, QC, H3A 2T7, Canada
[b] SIBA, Shanghai University of Finance and Economics, 777 Guoding Road, Shanghai 200433, China

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ABSTRACT

Blackwell (1951, 1953) proposes an informativeness ranking of experiments: experiment I is more Blackwell-informative than experiment II *if and only if* the value of experiment I is higher than that of experiment II for all expected-utility maximizers. Under commitment and reduction, our main theorem shows that Blackwell equivalence holds for all convex and strongly monotone preferences—i.e., uncertainty-averse preferences (Cerreia-Vioglio et al., 2011b), which nest most ambiguity-averse preferences commonly used in applications as special cases.

1. Introduction

In his seminal papers, Blackwell (1951, 1953) defines a partial ranking of experiments in which experiment I is more Blackwell-informative than experiment II if the latter is a garble of the former. Blackwell's theorem establishes that the value of experiment I is weakly higher than that of experiment II for all expected-utility maximizers and all sets of actions *if and only if* experiment I is more "Blackwell-informative" than experiment II. Motivated by experimental evidence,[1] an interesting open question is whether Blackwell's ranking is also appropriate for *ambiguity-averse* decision makers (DMs). Çelen (2012) studies the case for Gilboa and Schmeidler's (1989) maxmin expected utility (MEU) preferences.[2]

In this paper, we look for broader families of ambiguity preferences whose induced value of information characterizes the Blackwell ranking. We consider DMs who can commit to any (ex-ante) strategy and perceive ambiguity only in the states, while treating the information structures as objectively given. We show that, with relatively mild technical assumptions, most families of ambiguity preferences commonly used in applications—such as variational preferences

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* Corresponding author. Fax: +1 514 398 4938.
E-mail addresses: jian.li7@mcgill.ca (J. Li), zhoujj03001@gmail.com (J. Zhou).

[1] See, for example, Camerer and Weber (1992), Fox and Tversky (1995), Chow and Sarin (2001), Halevy (2007), and Abdellaoui et al. (2011).

[2] Heyen and Wiesenfarth (2015) propose a recursive calculation of the value of information; Gensbittel et al. (2015) consider ambiguous information structure and the no-commitment case. Both papers focus on the MEU case.

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(Maccheroni et al., 2006a), smooth ambiguity preferences (Klibanoff et al., 2005), multiplier preferences (Hansen and Sargent, 2001; Strzalecki, 2011), confidence preferences (Chateauneuf and Faro, 2009) and second-order expected utility (Grant et al., 2009)—can also induce a partial ranking of information that is equivalent to the Blackwell ranking. The largest such characterizing family we identify is the uncertainty-averse preferences (Cerreia-Vioglio et al., 2011b).

By focusing on the ex-ante decision value of information under commitment, we are able to show that Blackwell's ranking is very robust to introducing ambiguity aversion through almost all preference representations. This allows us to apply Blackwell's ranking with confidence in settings where ambiguity is prevalent and EU, or even MEU, axioms are violated. Furthermore, our proof technique highlights a link between Blackwell's equivalence and strongly monotone and convex preferences. Finally, we provide a unified framework for calculating the value of probabilistic information structure under ambiguity that is applicable to a broad range of economic problems without being restricted to any single preference representation.

The remainder of the paper is organized as follows. We describe notation in Section 2. Section 3 introduces uncertainty-averse preferences and the main assumptions. Section 4 presents the main theorem for uncertainty-averse preferences. Section 5 applies the main theorem to six well-known families of ambiguity preferences. Discussion is presented in Section 6. The appendix includes further discussion on extending the equivalence results to the no-commitment case for the maxmin expected utility and variational preferences, as well as direct proofs of smooth ambiguity preferences and second-order expected utility.

2. Notation

Our notation follows that of Çelen (2012). For any matrix $\mathbf{m}_{a\times b}$ of dimension $a\times b$, m_{ij} and $\mathbf{m}'$ denote the (i,j)th entry and the transpose of $\mathbf{m}$, respectively. The inner product of two matrices of the same dimension is defined as $\langle\mathbf{m},\mathbf{n}\rangle := \sum_i\sum_j m_{ij}n_{ij} = tr(\mathbf{m}'\mathbf{n})$. For any vector $\pi\in\mathbb{R}^n$, D^π denotes the diagonal matrix such that $D^\pi_{ii}=\pi_i$. Finally, $\mathbf{I}$ denotes the identity matrix.

Let $\Omega := \{\omega_1,\cdots,\omega_{|\Omega|}\}$ be a finite set of *states*. Denote by $\Delta(\Omega)$ the set of all priors on Ω, and $int(\Delta(\Omega))$ the set of priors with full support. Let $A := \{a_1,\cdots,a_{|A|}\}$ be a finite set of *actions* available to a DM.[3] A DM is characterized by a *utility function* or a *Von Neumann–Morgenstern (vNM) utility index* $u:\Omega\times A\mapsto\mathbb{R}$ and a *prior* $\pi\in\Delta(\Omega)$. We can construct a matrix $\mathbf{u}_{|\Omega|\times|A|}$ with entries $u_{\omega a}=u(\omega,a)$, for all $\omega\in\Omega, a\in A$.

Experiments, sometimes called *information structures*, are tuples $(\mathcal{S},\mathbf{p})$ and $(\mathcal{T},\mathbf{q})$, where $\mathcal{S} := \{s_1,\cdots,s_{|S|}\}$ and $\mathcal{T} := \{t_1,\cdots,t_{|T|}\}$ are sets of signals, and $\mathbf{p}_{|\Omega|\times|S|}$ and $\mathbf{q}_{|\Omega|\times|T|}$ are Markov matrices.[4] In particular, $p_{\omega s} := \Pr(s|\omega)$ for $s\in\mathcal{S}$ and $q_{\omega t} := \Pr(t|\omega)$ for $t\in\mathcal{T}$.

For a DM who observes a signal s from the experiment $(\mathcal{S},\mathbf{p})$, a strategy is a vector-valued mapping $\mathbf{f}:\mathcal{S}\mapsto\Delta(A)$. For each strategy $\mathbf{f}$, we define the matrix $\mathbf{f}_{|S|\times|A|}$, such that $(f_{j1},\cdots,f_{j|A|}) := f(s_j)$.[5] Similarly, we can define a strategy $\mathbf{g}:\mathcal{T}\mapsto\Delta(A)$. If a strategy maps every signal to the same (mixed) action $\mathbf{a}$ in $\Delta(A)$, it is identified with $\mathbf{a}$.

Blackwell (1951) defines the following ranking of two experiments.

Definition 1. An experiment $(\mathcal{S},\mathbf{p})$ is more Blackwell-informative than experiment $(\mathcal{T},\mathbf{q})$ if there exists a Markov matrix $\mathbf{r}$ such that $\mathbf{q}=\mathbf{pr}$.

The matrix $\mathbf{r}$ is also called the *garbling matrix*.

We incorporate ambiguity by considering an environment in which there is ambiguity about states in Ω, while the signal-generating process, described by the likelihood matrix $\mathbf{p}$, is treated as objectively given. By focusing on unambiguous signal likelihoods, we can relate the generalized value of signals under ambiguity to a clear ranking of their informational content. The examples below illustrate situations in which this assumption is natural.

Example 1 *(Partition).* In many economic and financial applications, information is represented by partitions of the state space. A finer partition is more Blackwell-informative. Specifically, if $\Omega=\{\omega_1,\omega_2,\omega_3,\omega_4\}$, then the partition $\{\{\omega_1\},\{\omega_2\},\{\omega_3,\omega_4\}\}$ is more informative than the partition $\{\{\omega_1,\omega_2\},\{\omega_3,\omega_4\}\}$. The likelihood and garbling matrices are

$$\underbrace{\begin{bmatrix}1&0\\1&0\\0&1\\0&1\end{bmatrix}}_{\mathbf{q}}=\underbrace{\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\\0&0&1\end{bmatrix}}_{\mathbf{p}}\underbrace{\begin{bmatrix}1&0\\1&0\\0&1\end{bmatrix}}_{\mathbf{r}}.$$

A DM may perceive ambiguity about the states. But conditional on the true state, a partitional signal structure unambiguously describes whether it belongs to each event in the partition.

[3] We assume that the number of available actions is larger than the number of signals.

[4] A matrix $\mathbf{m}$ is Markov if it is nonnegative and row stochastic—i.e., $m_{ij}\geq 0$ and $\sum_j m_{ij}=1$ for all i.

[5] Strategy $\mathbf{f}$ is a Markov matrix.