Overbidding and inefficiencies in multi-unit Vickrey auctions for normal goods

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A B S T R A C T

I examine bid behavior in uniform-price auctions and multi-unit Vickrey auctions, without the standard quasilinearity restriction on bidder preferences. Instead of assuming quasilinearity, I assume that bidders have weakly positive wealth effects, i.e. the goods are normal goods. My setting nests quasilinearity, but also allows for budget constraints, financial constraints, and risk aversion. I show that without the quasilinearity restriction, truthful reporting is not a dominant strategy in the Vickrey auction. Instead, bidders truthfully demand for their first unit and weakly overreport their demand for later units. The incentive to overreport demand means that the Vickrey auction is generally inefficient. This mirrors the well-known demand reduction results in uniform-price auctions. Moreover, the efficiency ranking of the two auctions is ambiguous. In fact, there are cases where the uniform-price auction is Pareto efficient and the Vickrey auction is Pareto inefficient, even if only one bidder has non-quasilinear preferences.

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1. Introduction

Governments often hold auctions for the stated goal of allocating resources to the public efficiently. Milton Friedman (1960) report on treasury auctions suggests that policy makers should use uniform-price auctions to efficiently allocate resources. In the decades that have followed, the uniform-price auction has been adopted to allocate treasury bills, electricity, and government-issued licenses. However, the demand-reduction result of Ausubel et al. (2014) shows that in uniform-price auctions, bidders have a strategic incentive to underreport their demands to the auctioneer. This demand reduction incentive implies that the uniform-price auction is inefficient. In contrast, in the benchmark private value auction setting, truthful reporting demand is a dominant strategy in the Vickrey auction and the auction yields an efficient allocation.

Yet, we rarely see the Vickrey auction implemented in practice. Ausubel and Milgrom (2006) provide four critiques of the Vickrey–Clarke–Groves (VCG) mechanism that suggest why VCG is rarely used in practice. However, their critiques are limited to heterogeneous good settings where bidders have complements preferences. On the sale of homogeneous goods, Rothkopf (2007) argues that Vickrey auctions are impractical because they are susceptible to collusion and perform

poorly when bidders have hard budget constraints.[2] However, the uniform-price auction also performs poorly in the face of budgets, and is similarly susceptible to collusion. Thus, it is unclear why homogeneous goods are frequently sold using the uniform-price auction instead of the Vickrey auction.

In this paper, I propose an explanation for why the uniform-price auction is used in favor of the Vickrey auction for the allocation of homogeneous goods. I remove the standard quasilinearity restriction on bidder preferences and consider a general preference domain that nests quasilinearity, but also allows risk aversion, financial constraints, and/or budgets. I show that without quasilinearity, the Vickrey auction loses its desirable incentive and efficiency properties. Instead of truthfully reporting their preferences, bidders truthfully report their demand for their first unit and overreport their demand for later units. We see that the Vickrey auction is generally inefficient because bidders have an incentive to misreport their preferences. This result mirrors the demand reduction result of Ausubel et al. (2014). I use my results on bid behavior to show that there is no clear efficiency ranking between the uniform-price auction and the Vickrey auction. In fact, there are cases where the uniform-price auction yields a Pareto efficient allocation of resources, and the Vickrey outcome is Pareto dominated. This can occur even if only one bidder has non-quasilinear preferences.

Prior work in the empirical auctions literature have compared the performance of Vickrey auctions and uniform-price auctions under the quasilinearity restriction. Most notably, Hortaçsu and Puller (2008) compare the performance of Vickrey auctions and uniform-price auctions in the Texas electricity spot market. They show that when we assume bidder preferences are quasilinear, there is a non-negligible efficiency loss associated with using the uniform-price auction instead of the Vickrey auction. Similarly, Fabra et al. (2002) study electricity markets and suggest policy makers use Vickrey auctions instead of uniform-price auctions, citing efficiency concerns.

However, the analysis in these papers assumes that bidders have quasilinear preferences. In auction theory, it is common to assume that bidders have quasilinear preferences for tractability. Yet, in many relevant economic environments, bidders are risk averse, have budgets, or face financing constraints.[3] Thus, allowing for non-quasilinear preferences provides a more complete description of bidder preferences.

I show that relaxing quasilinearity can reverse the efficiency ranking of the uniform-price and Vickrey auctions. I consider a setting where bidders have private values and multi-unit demands. By construction, in the Vickrey auction the price a bidder pays to acquire her first unit is lower than the price she pays for her second unit. If a bidder wins two units, then the payment rule is equivalent to having the bidder pay the (relatively higher) price for the second unit for both units, and then refunding her the difference in the two prices. This refund increases a bidders demand due to weakly positive wealth effects. Thus the bidder has an incentive to overreport her demand curve. I formalize this intuition, and show that any bid profile that understates a bidder's demand is weakly dominated.

For uniform-price auctions, I show that the intuition of Ausubel et al. (2014) holds when we remove the quasilinearity restriction. That is, any bid profile that overstates a bidder's demand and/or misreports her demand for her first unit is weakly dominated. However, bidders may have an incentive to underreport demand. In both cases, I form a partial characterization of bid behavior by looking at undominated strategies. While explicitly characterizing bid behavior is intractable, I show that considering only undominated strategies is sufficient for efficiency comparisons between the two auctions.

I use my bounds on bid behavior to develop an example that illustrates the ambiguity of the efficiency ranking the two auctions. In my example, only one bidder has non-quasilinear preferences, yet the outcome of the Vickrey auction is inefficient with positive probability. But, the outcome of the uniform-price auction is Pareto efficient with probability one.

The rest of the paper proceeds as follows. The remainder of the introduction relates my work to the auctions literature. Section 2 describes my model and provides a brief description of the uniform-price auction and the Vickrey auction. Section 3 proves results on bid behavior in both auctions. Section 4 concludes.

1.1. Related literature

Much of this prior literature about auctions assumes quasilinearity, but there is a literature study auctions with more general preferences. In the single unit environment, Matthews (1983) and Che and Gale (2006) show that risk aversion explains the experimental finding that first price auctions have higher revenues than second price auctions. Maskin and Riley (1984) and Baisa (2016) study the auction design problem when bidders do not have quasilinear preferences. Of this prior work, only Che and Gale (2006) and Baisa (2016) allow multidimensional heterogeneity across risk preferences and wealth effects like the setting studied here. Outside of the auctions literature, Garratt and Pycia (2014) discuss the efficient allocation of a normal good in a bilateral trade setting and show that we get qualitatively different results from those of Myerson and Satterthwaite (1983).

In the multi-unit auctions literature, most work that studies bidders with non-quasilinear preferences looks at the case where bidders have hard budget constraints. Recently, Dobzinski et al. (2012) showed that when bidders have private budgets, there is no dominant strategy mechanism that implements a Pareto efficient allocation and respects incentive compatibility when transfers are non-positive. In a related paper, Hafalir et al. (2012) study a modified Vickrey auctions for

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[2] Rothkopf (2007) also states Vickrey auctions require bidders to reveal valuable private information. Yet, Ausubel (2004) clinching auction, provides a dynamic implementation of the Vickrey auction with desirable privacy preservation properties. In addition, Rothkopf provides other critiques that are relevant only in heterogeneous good settings.

[3] See Che and Gale (1998) for examples on financing constraints, and Maskin (2000) for examples on budgets.