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Welfare criteria from choice: An axiomatic analysis ☆

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ABSTRACT

We propose an axiomatic approach to the problem of deriving a (linear) welfare ordering from a choice function. *Admissibility* requires that the ordering assigned to a rational choice function is the one that rationalizes it. *Neutrality* states that the solution covaries with permutations of the alternatives. *Persistence* stipulates that the ordering assigned to two choice functions is also assigned to every choice function in between.

We prove that these properties characterize the *sequential solution:* the best alternative is the alternative chosen from the universal set; the second best is the one chosen when the best alternative is removed; and so on. We also discuss some alternative axioms and solutions.

1. Introduction

The point of choice-based welfare analysis is to use information about choice behavior to draw inferences about welfare. Given an agent's *behavioral type*, which captures all observations about her choices from subsets of a universal set X, the problem is to determine a (potentially incomplete) welfare ranking of the alternatives in X. When an agent's behavior is fully rational, the standard answer is to adopt a *revealed preference approach*: the welfare ranking is simply the preference that is revealed to be maximized by the agent's choices. (To simplify the subsequent exposition, we sometimes equate a "behavioral type" with an "agent.")

In this paper, we extend choice-based welfare analysis to the general setting where agents may fail to be fully rational. In this setting, there is no real consensus about how choice behavior relates to preference. Instead, there is a patchwork of conflicting "bounded rationality" theories, none of which accommodates the full range of possible individual behavior. This makes it difficult to single out *one* way to assign welfare relations to agents. In moving to the general setting, another issue is that the domain of behavioral types expands significantly (beyond the set of "rational" agent types) while the range of welfare relations remains fixed. This dramatically increases the number of possible ways to assign welfare relations to agents; and it forces the same welfare relation to be assigned to a potentially wide variety of agents who exhibit different behavior.

To confront these issues directly, we consider the class of functions, called *solutions*, that assign a welfare relation to each behavioral type. Our approach is to impose axioms on such solutions. Of particular interest are "relational" axioms that

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formulate restrictions on the welfare relations assigned to different behavioral types. Such axioms impose coherence on welfare judgments across agents. Not only does this *axiomatic approach* provide a principled basis to evaluate and compare solutions but it also helps to hone in on a solution. By insisting on a specific form of coherence, one can narrow the range of potential solutions tremendously. We believe that this leads to sound policy. Since our ultimate goal is to develop individual welfare measures that can be aggregated to evaluate social welfare, it is essential to make coherent welfare judgments across agents: this ensures that the resulting social welfare judgments are meaningful.

Our axiomatic approach to welfare is quite flexible. In principle, the notion of a solution can be tailored to fit the type of input choice data observed and the kind of output relation required for policy making. For the sake of convenience, the current paper focuses on the simplest setting. For a solution in this *canonical setting*, the domain consists of all possible choice functions on X and the range consists of all possible (linear) orderings on X. In Section 5.3, we discuss how to extend our approach to a variety of non-canonical settings.

We consider three natural axioms in the canonical setting: admissibility, neutrality and persistence. *Admissibility* requires that the ordering assigned to a rational choice function must be the one that rationalizes it. In turn, *neutrality* states that the solution covaries with respect to permutations of the alternatives. Finally, *persistence* stipulates that if the same ordering is assigned to two choice functions, then it is assigned to any choice function *in between* – that is, any choice function which, from each set, selects one of the alternatives chosen by the other two.

Our main result shows that these three axioms uniquely determine a solution that is straightforward to compute from choice behavior. According to this *sequential solution*, the best alternative is the one chosen from the universal set X; the second best alternative is the one chosen when the best alternative is removed from X; and so on.

In our view, the result illustrates the power of the axiomatic approach. First, it shows that a few natural properties can uniquely determine a simple solution even though the scope of possibilities is quite formidable.[1] In addition, it shows that axioms can combine in unexpected ways. Indeed, the solution that we characterize is inherently sequential even though none of the axioms has this feature. What is more, it completes the welfare relation proposed by Bernheim and Rangel (2009) even though none of our axioms is clearly related to their approach.

In accordance with the axiomatic method, we feel that the merits of a solution should be judged on the basis of its axiomatic foundations. Having said this, we are cautious about interpreting our result as conclusive support for the sequential solution. While there are compelling reasons to insist on each of our three axioms, there are also reasons (discussed at greater length in Section 2) to take issue with each. With this in mind, we are inclined to view our work as the first step towards a compelling theory of welfare based on the axiomatic approach. In Sections 4 and 6, we briefly touch on some of the most important issues that we feel still need to be resolved.

Before turning to the related literature, we point out that the relevance of our characterization extends beyond welfare analysis. The sequential solution enjoys a certain "folk" status in the literature, having been used in a variety of different contexts (see Marschak, 1955; Arrow and Raynaud 1986, Ch. 7 or Moulin 1988, Exercise 11.9, for example). Clearly, its prevalence owes much to its simplicity as a method for extracting an ordering from choice data. Our result provides an independent normative justification of this solution. Whether this is ultimately compelling will, of course, depend on the specific purpose intended for the derived ordering.

Related literature. The problem of choice-based welfare evaluation for boundedly rational agents has attracted considerable attention in the recent literature. By far the most popular suggestion is to extend the approach used in the standard setting – by defining revealed preference criteria appropriate for agents in the general setting. For proponents of this approach, the debate centers around which notion of revealed preference is best suited to the task (see Manzini and Mariotti, 2014 for a recent survey). Some, like Rubinstein and Salant (2012), advocate a *model-specific* approach where the revealed preference criteria are derived from a specific model of bounded rationality. Others, like Bernheim and Rangel (2009), favor a *model-free* approach. Specifically, they propose the following welfare criterion: an agent is better off with alternative x than alternative y if the agent never chooses y when x is available.[2] While certainly intuitive, this Pareto-like criterion does not rely on an explicit model of behavior.

Conceptually, our axiomatic approach is quite distinct from this revealed preference approach. Instead of treating each agent in isolation, it views the set of agents as a whole – with the explicit goal of making coherent welfare judgments across agents. Among the few other papers in the literature that do not follow a revealed preference approach, two are worth mentioning.

In the first paper, Nishimura (2014) axiomatizes a function, called the *transitive core*, that assigns a reflexive and transitive (but potentially incomplete) welfare relation to each complete (but potentially cyclic) binary relation on X. The fundamental difference from our work is that Nishimura does not work with choice data directly. Instead, he considers a binary relation that can be defined (or derived) from choice data. This situates his work much closer to the vast literature on extracting orderings from tournaments (see Bouyssou, 2004 for a survey).

In the second paper, Apesteguia and Ballester (2015) axiomatize an *inconsistency index* which, for each behavioral type, measures the "swaps difference" from the closest orderings on X. Implicitly, their approach defines a multivalued solution:

[1] For $|X| = n$ in the canonical setting, there are exactly $n!^{K(n)}$ solutions where $K(n) := \prod_{k=1}^{n} k^{\binom{n}{k}}$.

[2] Independently, Green and Hojman (2009) propose the same welfare criterion.