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Note

A note on optimal cheap talk equilibria in a discrete state space

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ABSTRACT

A discrete version of Crawford and Sobel's (1982) cheap talk model is considered. Unlike in the continuous case, limiting attention to partitional equilibria is with loss of generality. The need to consider equilibria that are non-partitional complicates the analysis. It is shown that if utility functions are concave and the sender is upwardly biased, then the receiver's optimal equilibrium is necessarily partitional. Based on this result, a simple characterization of the optimal equilibrium for the discrete uniform quadratic case is proposed.

1. Introduction

The cheap talk model of Crawford and Sobel (1982) (henceforth CS) is central to economic theory. Numerous different specifications of this model were developed in an attempt to improve our understanding of various situations of strategic communication.[1] Most of the literature focuses on the original, continuous version of the model. Moreover, due to its technical tractability, the uniform quadratic specification has become the canonical model in many theoretical and applied papers.[2]

Many situations fit well into the continuous model but sometimes it may be natural to consider a discrete version of the model. There are situations in which the corresponding state space is inherently discrete, and sometimes the sender's information can be endogenously organized in coarse and discrete information sets (see, e.g., Fischer and Stocken, 2001, Ivanov, 2010).

In the CS model, what matters is the conveyed information. A common practice for the continuous version of the model is to identify any equilibrium with the receiver's induced information *partition*. Conceptually, CS equilibria in a discrete state space are similar to those of the continuous case. However, in the discrete version of the model, the sender's types have a strictly positive measure. As a result, limiting attention to partitional equilibria is with loss of generality and the need to account for non-partitional equilibria complicates the analysis.

In this paper, I consider a discrete version of CS cheap talk game. More precisely, I study optimal equilibria in the case where the state space is discrete, while the receiver's action space is continuous as in CS. I begin with providing an example where the (ex-ante) most desired equilibrium, from the receiver's perspective, is non-partitional. Next, I show that if, at each state of the world, the players' utility functions are concave and the sender is upwardly biased (defined below), then the

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[1] Sobel (2010) provides a comprehensive review of the communication literature.

[2] See, e.g., Blume et al. (2007), Goltsman et al. (2009), Ivanov (2010), and Krishna and Morgan (2004).

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receiver's most preferred (ex ante) equilibrium is necessarily partitional (Proposition 1). As a special case, this class of games includes the discrete version of the leading uniform quadratic specification of the model where, ex ante, the players rank different equilibria identically. I conclude by providing a simple method for calculating the optimal equilibrium for this environment (Proposition 2).

2. Model

There are two players, sender and receiver, $N = \{S, R\}$. A state $\theta \in \Theta = \{1, 2, .., n\}$ is distributed according to a common prior P. First, the sender privately observes her "type" θ. Then, she submits a cheap talk message $m \in M$ after which the receiver chooses an action[3] $a \in \mathbb{R}$. Player i's preferences are given by a supermodular utility function $u^i(a, \theta)$. For each θ, $u^i(\cdot, \theta)$ is concave, and there exists an action $a^i(\theta) \in \mathbb{R}$ that is most desirable from the perspective of player i at state θ.

The sender chooses a reporting strategy $m : \Theta \to \Delta(M)$, and the receiver chooses an action rule $a : m \to \mathbb{R}$. The solution concept is a Bayesian–Nash equilibrium. Under the assumption that $u^R(\cdot, \theta)$ is concave, for every belief $\mu \in \Delta(\Theta)$ there is a unique optimal action $a^R(\mu)$ for the receiver. Thus, the assumption that the receiver chooses a deterministic action is without loss of generality. As usual, every (informational) outcome that is consistent with equilibrium can be reproduced with a sender's strategy that has a full support. Accordingly, I focus on the content of the reports in equilibrium without specifying the exact form of information transmission.

A sender's report induces a receiver's belief. More generally, a sender's strategy induces a belief structure $\tilde{\mu} \in \Delta(\Delta(\Theta))$, that is, a distribution *over beliefs* over the states.

A belief structure $\tilde{\mu}$ is *monotone* if for every $\mu, \eta \in supp(\tilde{\mu})$, either $\theta_\mu \geq \theta_\eta$ for all $\theta_\mu \in supp(\mu)$ and $\theta_\eta \in supp(\eta)$, or $\theta_\mu \leq \theta_\eta$ for all $\theta_\mu \in supp(\mu)$ and $\theta_\eta \in supp(\eta)$. A belief structure $\tilde{\mu}$ is *partitional* if for every $\theta \in \Theta$ and two distinct elements $\mu, \eta \in supp(\tilde{\mu})$, $\mu(\theta) > 0$ implies that $\eta(\theta) = 0$. An equilibrium is monotone (partitional) if it induces a monotone (partitional) receiver's belief structure. An equilibrium that is monotone and partitional induces an *interval partition* $\{J_1, ..., J_k\}$ of $\{1, ..., n\}$, that is, for every two integers x and y such that $1 \leq x < y \leq n$, if $x \in J_i$ then $y \in J_j$ for some $j \geq i$.

In other words, monotonicity requires that whenever two sender types $\theta_\mu > \theta_\eta$ send messages inducing distinct receiver beliefs μ and η, it must be that the lowest type $\underline{\theta}_\mu$ in the support of μ weakly exceeds the highest type $\bar{\theta}_\eta$ in the support of η. If $\underline{\theta}_\mu > \bar{\theta}_\eta$ for all such equilibrium beliefs μ and η, then the equilibrium is also partitional; whereas monotonic non-partitional equilibria can arise if there is a type $\underline{\theta}_\mu = \bar{\theta}_\eta$ who mixes between inducing the beliefs μ and η.

An equilibrium e' generates *redundant* information for the receiver if there are two distinct beliefs $\mu, \eta \in supp(\tilde{\mu}_{e'})$ such that $a^R(\mu) = a^R(\eta)$. It is immediate that for every such e' there exists a payoff-equivalent *non-redundant* equilibrium e that spares the receiver all the redundant information. Throughout, I focus on non-redundant equilibria.

As in CS, a direct consequence of non-redundancy and supermodularity is that every equilibrium is monotone. However, the following example shows that non-partitional equilibria not only exist, but can be most desired.

Example 1. Let $\Theta = \{1, 2, 3\}$. Assume that $p(\theta = 2) = \alpha$ and $p(\theta = 1) = p(\theta = 3) = \frac{1-\alpha}{2}$, where $\frac{1}{2} < \alpha < 1$. The players' ideal points are presented in Table 1. Assume that the players are minimizing the (expected) quadratic loss (that is, $u^i(a, \theta) = -(a^i(\theta) - a)^2$).

Table 1
Ideal receiver's actions for both players at different states of the world.

	$\theta = 1$	$\theta = 2$	$\theta = 3$
$a^R(\theta)$	1	2	3
$a^S(\theta)$	$1\frac{2}{3}$	2	$2\frac{1}{3}$

It is immediate that if the receiver holds a belief μ, his optimal action is $a^R(\mu) = E_\mu[\theta]$. As always, "babbling" (a reporting strategy according to which the sender submits a random message, uncorrelated with her private information) is consistent with equilibrium. I now show that this is the unique (non-redundant) partitional equilibrium in this case.

A fully separating equilibrium does not exist as every sender's type $\theta \neq 2$ would then have a profitable deviation to pretend that $\theta = 2$. If $\{1, 2\}$ is an element of the resulting receiver's partition, the induced actions are $a^R(\{1, 2\}) = E[\theta|\theta \in \{1, 2\}] > 1\frac{2}{3}$ and $a^R(\{3\}) = 3$. Thus, the sender's type $\theta = 3$ prefers $a^R(\{1, 2\})$ to $a^R(\{3\})$, and so the partition $\{\{1, 2\}, \{3\}\}$ is inconsistent with equilibrium. Similarly, $\{\{1\}, \{2, 3\}\}$ is inconsistent with equilibrium as well.

I now construct an informative equilibrium in which two actions a_l and a_h ($a_l < a_h$) are induced. Hence, this equilibrium is preferred by the receiver to the "babbling equilibrium."

If $\theta = 1$ the sender induces a_l with probability 1. If $\theta = 3$ the sender induces a_h with probability 1. In the case of $\theta = 2$, the sender induces each of the actions with probability $\frac{1}{2}$. It follows that $a_l = 1 + \alpha$ and $a_h = 3 - \alpha$. The action a_l (a_h) is

[3] It is assumed that M is rich enough so that it does not restrict the sender's reporting flexibility.