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Note

Reduced form implementation for environments with value interdependencies

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ABSTRACT

We provide a unified and simple treatment of reduced-form implementation for general social choice problems and extend it to environments with value interdependencies. We employ the geometric approach developed by Goeree and Kushnir (2016) to characterize the set of feasible interim agent values (agent utilities excluding transfers) by deriving the analytical expression of its support function. As an application, we use the reduced-form implementation to analyze second-best mechanisms in environments with value interdependencies.

1. Introduction

A typical problem of a mechanism designer can be expressed as a maximization of some objective over the set of feasible and incentive compatible allocation rules and transfers. In standard settings incentive compatibility identifies transfers from interim allocation probabilities up to a constant (Milgrom and Segal, 2002). This reduces the problem of the mechanism designer to an optimization over interim allocation probabilities only, i.e. *reduced form* problem.

To study the reduced form problem one should be able to identify the set of feasible interim allocation probabilities. Matthews (1984) first conjectured the set of inequalities characterizing this set in symmetric single-object auctions. These inequalities, as a necessary condition, were first used by Maskin and Riley (1984) and Matthews (1983) to analyze single-object auctions with risk-averse bidders. The sufficiency of these inequalities were subsequently proven by Border (1991) who then extended the result to asymmetric auction environments (see Border, 2007).[2] This characterization was used in numerous papers in the past and attracted attention of many recent papers.[3]

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[2] For recent developments in reduced form auctions see Vohra (2011), Mierendorff (2011), Alaei et al. (2012), Cai et al. (2012), Che et al. (2013), Hart and Reny (2015), and Goeree and Kushnir (2016).

[3] See Armstrong (2000), Brusco and Lopomo (2002), Morand and Thomas (2006), Manelli and Vincent (2010), Asker and Cantillon (2010), Belloni et al. (2010), Hörner and Samuelson (2011), Miralles (2012), Pai and Vohra (2013, 2014a, 2014b), Pai (2014), Mierendorff (2014).

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In this paper we provide a unified and simple treatment of reduced form implementation for general social choice problems and extend it to environments with value interdependencies. In the presence of value interdependencies, interim allocation probabilities alone do not determine interim agent utilities. To account for them, we characterize feasible interim agent values, i.e. interim agent utilities excluding transfers.

To characterize the set of feasible interim agent values we use the geometric approach developed by Goeree and Kushnir (2016) who exploit the one-to-one relation between a convex closed set and its support function. To obtain the support function for interim agent values we exploit the fact that they are a linear transformation of ex post allocations. For each possible type profile, feasible ex post allocations form a simplex for which the support function is well-known. Utilizing the rule how the support function transforms under a linear transformation we then determine the support function for feasible interim values. Finally, we use the duality from convex analysis to recover the inequalities characterizing the set of feasible interim values.

To illustrate our main result we apply reduced form implementation to study auction environments with multiple agents and linear value interdependencies. Maskin (1992) and Dasgupta and Maskin (2000) show that for large value interdependencies the first-best social surplus cannot be implemented with incentive compatible mechanisms. Using the reduced-form implementation results we analyze the second-best mechanism for these environments. We show that the optimal Bayesian and dominant strategy incentive compatible mechanisms lead to the same level of social surplus if the object has to be allocated to agents. If the object does not have to be allocated to agents we provide a condition on the level of interdependencies when both implementation concepts achieve the same level of social surplus.

The paper proceeds as follows. Section 2 presents a social choice model. We derive the support function for the feasible set of interim agent values in Section 3. Section 4 analyzes second-best mechanisms in the environments with value interdependencies. Section 5 concludes.

2. Social choice model

We consider an environment with a finite set $\mathcal{I} = \{1, 2, ..., I\}$ of agents and a finite set $\mathcal{K} = \{1, 2, ..., K\}$ of social alternatives. Agent $i \in \mathcal{I}$ has a one-dimensional type x_i with finite support $X_i = \{x_i^1, \ldots, x_i^{N_i}\} \subset \mathbb{R}_+$.[4] We also denote the profile of all agent types as $\mathbf{x} = (x_1 \ldots, x_I)$ with support $X = \prod_i X_i$. We allow for correlation in types and denote their joint probability distribution as $f(\mathbf{x})$. Agent values are interdependent: when alternative k is selected and the profile of agent types is $\mathbf{x}$ agent i's value equals $v_i^k(\mathbf{x})$.

A direct mechanism can be characterized by $K + I$ functions, $\{q^k(\mathbf{x})\}_{k\in\mathcal{K}}$ and $\{t_i(\mathbf{x})\}_{i\in\mathcal{I}}$, where $q^k(\mathbf{x})$ is the probability that alternative k is selected and $t_i(\mathbf{x}) \in \mathbb{R}$ is agent i's transfer. We denote then agent i's ex post values as $v_i(\mathbf{x}) = \sum_{k\in\mathcal{K}} v_i^k(\mathbf{x}) q^k(\mathbf{x})$ and interim expected values as $V_i(x_i) = \sum_{\mathbf{x}_{-i}} f_{-i}(\mathbf{x}_{-i}|x_i) v_i(\mathbf{x})$, where $f_{-i}(\mathbf{x}_{-i}|x_i)$ is the distribution of other agent types $\mathbf{x}_{-i}$ conditional on x_i. When agents report their types truthfully, agent i's ex post utility equals $u_i(\mathbf{x}) = v_i(\mathbf{x}) + t_i(\mathbf{x})$, and his interim expected utility equals $U_i(x_i) = V_i(x_i) + T_i(x_i)$, where $T_i(x_i) = \sum_{\mathbf{x}_{-i}} f_{-i}(\mathbf{x}_{-i}|x_i) t_i(\mathbf{x})$.

We heavily exploit the notion of the *support function* $\mathcal{S}^C : \mathbb{R}^n \to \mathbb{R} \cup \{+\infty\}$ of a closed convex set $C \subset \mathbb{R}^n$, which is defined as

$$\mathcal{S}^C(\mathbf{w}) = \sup\{\mathbf{v} \cdot \mathbf{w} \,|\, \mathbf{v} \in C\}, \tag{1}$$

with the inner product $\mathbf{v} \cdot \mathbf{w} = \sum_{j=1}^n v_j w_j$. Support functions have three important properties that we outline below (for more details, see Rockafellar, 1997). First, there is the one-to-one relation between the support function and the corresponding convex set: given the support function S^C one can always recover the corresponding set as $C = \left\{\mathbf{v} \in \mathbb{R}^n \,|\, \mathbf{v} \cdot \mathbf{w} \leq \mathcal{S}^C(\mathbf{w}),\ \forall \mathbf{w} \in \mathbb{R}^n\right\}$. Second, the support function for a Cartesian product of sets $C_1, C_2 \subset \mathbb{R}^n$ equals the sum of the support functions: $S^{C_1 \times C_2}(\mathbf{w}') = S^{C_1}(\mathbf{w}_1) + S^{C_2}(\mathbf{w}_2)$, where $\mathbf{w}' = (\mathbf{w}_1, \mathbf{w}_2) \in \mathbb{R}^{2n}$, which directly follows from definition (1). Finally, the support function straightforwardly changes under a linear transformation. In particular, consider a linear transformation $A : \mathbb{R}^n \to \mathbb{R}^m$ and a closed convex set $C \subset \mathbb{R}^n$. Recall a basic property of the inner product $Aq \cdot w = q \cdot A^T w$, where $q \in C$ and A^T is the transpose of A. Therefore, the support function corresponding to image AC of linear transformation A equals $\mathcal{S}^{AC}(\mathbf{w}) = \mathcal{S}^C(A^T \mathbf{w})$, where $\mathbf{w} \in \mathbb{R}^n$.

3. Reduced form implementation

This section presents our main result that presents an analytical expression of the support function associated with the set of feasible interim values $V_i(x_i)$. To accomplish this goal we use the novel geometric approach developed recently by Goeree and Kushnir (2016).

For a given type profile $\mathbf{x}$, let us consider allocation $\{q^k(\mathbf{x})\}_{k\in\mathcal{K}}$ defining a $(K-1)$-dimensional simplex, i.e. $q^k(\mathbf{x}) \geq 0$ and $\sum_{k\in\mathcal{K}} q^k(\mathbf{x}) = 1$. If we consider 2-simplex $\{q^k \geq 0,\ k = 1, 2, 3;\ q^1 + q^2 + q^3 = 1\}$, depicted in Fig. 1, it is straightforward to verify that the inner product $\mathbf{q} \cdot \mathbf{w}$ achieves the maximum at one of the extreme points $(0, 0, 1)$, $(0, 1, 0)$, or $(1, 0, 0)$ of the simplex resulting in support function $\mathcal{S}^{2\text{-}simplex}(\mathbf{w}) = \max\left(w^1, w^2, w^3\right)$. This expression straightforwardly generalizes to

[4] Our main result, Theorem 1, also holds without any changes in environments with multi-dimensional types.