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Identifying subjective beliefs in subjective state space models

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ABSTRACT

This paper takes the Dekel, Lipman, and Rustichini (2001) framework, in which subjective beliefs over subjective states cannot be identified, and proves a conjecture made in their paper: if the Bernoulli utility functions are additively separable and one of the terms is state-independent, then beliefs are uniquely identified. The main departure from existing literature is that beliefs are identified without imposing extra objective elements into the model.

1. Introduction

Expected utility models of decision making under uncertainty focus on three objects: tastes, represented via a Bernoulli utility function, possible contingencies or *states*, represented by an abstract set of labels, and beliefs, represented by a probability distribution over the set of labels. Bernoulli utilities, labels, and probabilities are useful representations of tastes, possible contingencies, and beliefs only if they are uniquely identified from choice behavior. We do not have a model where Bernoulli utilities, state spaces, and beliefs are all simultaneously identified from preferences. Indeed, a state-dependent Bernoulli utility function is crucial for identifying subjective states, but following standard arguments, this state-dependence implies that beliefs are not uniquely identified (see Kreps, 1979; Dekel, Lipman, and Rustichini, 2001, henceforth DLR; and Anscombe and Aumann, 1963, henceforth AA). This paper takes a conjecture from DLR—that if the outcome space is two-dimensional, Bernoulli utilities are additively separable, and one of the terms is state-independent, then beliefs are uniquely identified—and proves that the conjecture is true. We provide axioms to guarantee that the resulting representation is additively separable with one state-independent term, and the identification procedure is a simple extension of the AA technique.

The DLR framework is relevant for game theoretic applications where a player must make initial commitments without knowing how these will impact his future utility. Two situations where this problem arises are optimal trade-offs between commitment and flexibility in a consumption-savings model (see, for example, Amador, Werning, and Angeletos, 2006), and game theoretic models of unforeseen contingencies, generally related to asset pricing or incomplete contracts (see,

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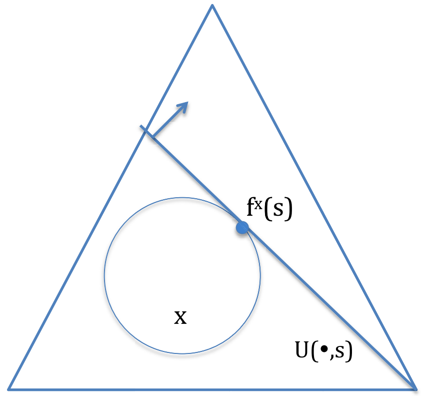

Fig. 1. At state s the decision maker chooses element $f^x(s)$ from menu x.

for example, Kraus and Sagi, 2006 for asset pricing applications, or Maskin and Tirole, 1999 for incomplete contracting applications). More generally, in extensive form games where a player moves more than once, every move he makes is a commitment to a subset of strategies, and if a player must make one such commitment prior to learning information that is relevant to his von Neumann–Morgenstern utility (for example, he might be unaware of some moves his opponent must make) then such a game falls within the DLR framework: the choice of a menu is the choice of an initial commitment in the game, the subjective states are the information he expects to learn within the game, and the choice of an element out of a menu is his choice of a strategy. Furthermore, the quasilinear representation we are after is relevant for games where payments are made or received in exchange for goods or services.

Identification of beliefs is important for applications that want to consider comparative statics exercises on the beliefs a decision maker holds about the likelihood with which the different states occur. Consider a DLR model where a representation of preferences is indexed by the three objects mentioned above: a state-dependent Bernoulli utility function, U, a set of labels, S, and a probability distribution over those states, μ. Then, for any measure ν over S that is absolutely continuous with respect to μ, there is a Bernoulli utility function $\hat{U}$ such that $(\hat{U}, S, \nu)$ also indexes a representation of the same preferences (see the example in Section 2 for details). Since μ and ν may be unrelated one cannot claim that they represent the "beliefs" a decision maker holds about the likelihood with which events occur. For example, if μ and ν are such that for some $s, s' \in S$ we have that $\mu(s) > \mu(s')$ and $\nu(s') > \nu(s)$, we cannot claim that the DM believes s is more likely than s' whenever we represents preferences with (U, S, μ), but he believes s' is more likely than s whenever we represent preferences with $(\hat{U}, S, \nu)$. Therefore, probabilities do not represent "beliefs", so asking how some variables change when we change μ, and interpreting these changes as the changes that occur when the DM's beliefs change, is incorrect. However, in an environment such as ours, where μ is uniquely identified from preferences, μ may indeed be interpreted as the "beliefs" a DM holds about the likelihood with which different events occur, so comparative statics on μ may indeed be interpreted as comparative statics on a DM's "beliefs".

The paper is organized as follows. Section 1.1 informally discusses why, even with additive separability and state independence of the Bernoulli utility function, belief identification is non-trivial; it also (informally) presents the axioms required to obtain a representation with an additive separable Bernoulli utility function that is state-independent in one dimension. Section 1.2 comments on the most closely related paper (Sadowski, 2013). Section 2 provides an overview of the Dekel, Lipman, and Rustichini's (2001) model, while Section 3 introduces the novel axioms and states the representation and identification theorems. Section 4 concludes.

1.1. Informal discussion of results

Identifying subjective beliefs is not trivial, even with additive separability and state-independence on one dimension, because models of menu choice are equivalent to models of AA acts where the modeler is restricted in the set of acts he can offer the decision maker. In our setup, the decision maker considers a set of alternatives, B, and the objects of choice are menus of lotteries over B, i.e. subsets $x \subset \Delta(B)$; states are identified with von Neumann–Morgenstern (vNM) utilities over $\Delta(B)$ (see Section 2 for details). For any menu x, and any state s, define $f^x(s)$ as the lottery in x that maximizes utility at s. This then defines an AA act (see Fig. 1 above), but clearly not all acts can be spanned in this way. In general, this lack of richness precludes belief identification. For example, if the modeler can only offer acts that are constant over some set of states E, it is impossible to identify the subjective probability of any event $E' \subset E$.

In our setup, when state spaces are finite, this lack of richness is not a problem and beliefs can easily be identified, as illustrated by Fig. 2 below for the case of two subjective states. As an illustration, note that since outcomes are two-dimensional (we call these dimensions M and Z) and since the Bernoulli utility function is additively separable over these dimensions, then any menu x is equivalent to an act $f^x = (f^x_M, f^x_Z)$ that maps states into lotteries over each dimension. In the figure, let x be the menu where the decision maker can choose either β_Z or β'_Z on the Z-dimension, but always gets β_M