Games and Economic Behavior 95 (2016) 41–46

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Note

Characterizing minimal impartial rules for awarding prizes ☆

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ARTICLE INFO

Article history:
Received 1 March 2015
Available online 17 December 2015

JEL classification:
D63
D71

Keywords:
Institutional design
Award rules
Conflict of interest
Impartiality
Plurality correspondence

ABSTRACT

We study the problem of selecting prize winners from a group of experts when each expert nominates another expert for the prize. A nomination rule determines the set of winners on the basis of the profile of nominations; the rule is impartial if one's nomination never influences one's own chance of winning the prize. In this paper, we consider impartial, anonymous, symmetric, and monotonic nomination rules and characterize the set of all minimal such rules. We show that the set consists of exactly one nomination rule: a natural variant of the plurality correspondence called plurality with runners-up.

1. Introduction

Suppose that a foundation is considering awarding a prize to one or more members of a group of experts whose activities advance the public interest. The foundation's leader wishes to select members who most deserve the prize, but he cannot do so by himself because he lacks the expertise needed to evaluate their merits. Given that situation, this paper considers the design of award rules that base the selection of winners on experts' views. In particular, we study *nomination rules* that ask each expert to nominate one other expert for the prize; the set of winners is then determined based on the profile of nominations. The challenge of this approach is that conflicts of interest might be created among selfish experts. In particular, a person caring only about her own winning might corrupt her nomination when there is a chance that she can influence her own likelihood of taking the prize. We are thus interested in nomination rules that create no such conflict of interest among selfish experts, and study those satisfying an axiom called *impartiality*. A nomination rule is impartial if it determines each person's winning independently of her nomination; a selfish person thus has no chance to influence her own winning when the rule satisfies impartiality.

The aim of this paper is to identify reasonable impartial nomination rules among those satisfying three additional axioms: *anonymity*, *symmetry*, and *monotonicity*. Anonymity requires that an exchange of nominations between two people do

☆ The author is indebted to Shinji Ohseto, an associated editor, and two anonymous referees for their helpful suggestions and useful comments. He also thanks Kazuhiko Hashimoto, Ryo Horii, Shigemi Izumida, Takashi Kunimoto, Mitsunobu Miyake, Shuhei Morimoto, Ryoichi Nagahisa, Satoshi Nakada, Shuhei Otani, Toyotaka Sakai, Shigehiro Serizawa, Koichi Tadenuma, Takashi Ui, Takuma Wakayama, Zhen Zhao, participants at the 2015 Spring Meeting of the Japanese Economic Association, and seminar participants at Hitotsubashi University, Osaka University, and Tohoku University for their insightful comments. An earlier version of this paper is awarded the 17th Osaka University Institute of Social and Economic Research/Moriguchi Prize. Comments from two anonymous referees of this prize have also been helpful. The research was partially supported by Grant-in-Aid for JSPS fellows (15J04747) from Japan Society for the Promotion of Science.

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http://dx.doi.org/10.1016/j.geb.2015.12.005
0899-8256/

not affect the winning of any other person. Symmetry requires the determination of the set of winners to be independent of the indexes of people. Monotonicity requires that any subset of winners be included in the new set of winners when each member in the subset obtains an additional nomination from another person.

Now, consider the nomination rule under which all people are always chosen as the winners. Although satisfying the three axioms and being impartial, we cannot describe such a nomination rule as reasonable. By always selecting too many winners, without examining their qualifications, it might degrade the prestige of the prize, which the foundation aims to maintain. It might also undermine the social practice of competition. These arguments confirm that it is desirable for a nomination rule to select winners as strictly as possible, leading us to the question of which nomination rules are optimal in this sense subject to all the four axioms.

In this paper, we obtain an explicit answer to this question by exploring *minimal* nomination rules among those satisfying the four axioms. We define a nomination rule satisfying the four axioms as "minimal" if one cannot make a further refinement to the nomination rule while still preserving the four axioms, i.e., if there is no other nomination rule that satisfies the four axioms while assigning to every profile of nominations a set of winners that is smaller, compared by inclusion, than that assigned by the nomination rule under consideration. The result will thus characterize the set of all minimal nomination rules satisfying the four axioms. We show that *plurality with runners-up* (Tamura and Ohseto, 2014) *is the only minimal nomination rule satisfying impartiality, anonymity, symmetry, and monotonicity*. Plurality with runners-up is a natural variant of the plurality correspondence. Indeed, the set of winners is always that of plurality winners except when there is a sole plurality winner who defeats the runners-up by only one point; in this case, a runner-up who nominates the sole plurality winner also wins.

This paper is the first, to our knowledge, to establish a characterization result in the context of impartial nomination rules. Holzman and Moulin (2013) begin this area of study with "single-valued" nomination rules and propose interesting impartial rules called the *partition methods*. Instead of characterizing these partition methods, they establish two impossibility results regarding single-valued impartial nomination rules; one of these states that no such rule simultaneously satisfies two desirable axioms which they call "positive unanimity" and "negative unanimity".[1] Tamura and Ohseto (2014) then allow rules to be "multi-valued," as is done in this paper, focusing on discussing whether Holzman and Moulin's impossibility results hold in a more general class of multi-valued nomination rules. By constructing the "plurality with runners-up" correspondence, they show that there exists an impartial rule meeting positive and negative unanimity when at least four people are involved.

In the closely related context of "impartial division rules," a characterization result has already been established. de Clippel et al. (2008) study the problem of dividing a surplus among a group of partners when each partner represents her subjective opinion about the relative contributions of the others to the surplus. A division rule determines the division of the surplus on the basis of the profile of opinions, and impartiality requires that the share of the surplus each person receives be independent of her own opinion. For situations of four or more partners, the authors propose an infinite family of impartial rules that aggregate the opinions of the partners in a highly natural way. They then characterize that family by employing several reasonable axioms. A clear difference exists between de Clippel et al.'s result and ours: they characterize the whole class of rules meeting their axioms, whereas we characterize only the minimal rules satisfying our axioms. Nevertheless, this difference does not degrade the importance of our result; as explained above, investigating minimal nomination rules is itself meaningful in our context.

The rest of the paper is organized as follows. In Section 2, we introduce the model and the axioms. In Section 3, we state and prove the result. In Section 4, we offer concluding remarks.

2. Model and axioms

Let $N = \{1, \ldots, n\}$ $(n \geq 3)$ be the set of people. For each $i \in N$, let $x_i \in N \setminus \{i\}$ denote i's *nomination*. If $x_i = j$, we say that i nominates j. A list $x = (x_i)_{i \in N}$ is called a *nomination profile*. Let N_{-}^{N} denote the set of all nomination profiles. For each $x \in N_{-}^{N}$ and each $i_1, \ldots, i_m \in N$, where $m = 1, \ldots, n$, we sometimes write x for $(x_{\{i_1,\ldots,i_m\}}, x_{N \setminus \{i_1,\ldots,i_m\}})$ to distinguish the nominations of $i_1, \ldots, i_m$ from those of the others in x. For simplicity of notation, we often use (x_i, x_{-i}) instead of $(x_{\{i\}}, x_{N \setminus \{i\}})$. A *nomination rule* is a correspondence $\varphi : N_{-}^{N} \to 2^N \setminus \{\emptyset\}$ that assigns a non-empty subset of people, which we mention as the set of winners, to each nomination profile.

We next introduce four axioms that we impose on nomination rules. First, as our central axiom, *impartiality* requires that one's nomination never influences one's own winning.

Impartiality: for all $x \in N_{-}^{N}$, all $i \in N$, and all $x'_i \in N \setminus \{i\}$,

$$i \in \varphi(x_i, x_{-i}) \Leftrightarrow i \in \varphi(x'_i, x_{-i}).$$

Second, we consider *anonymity* which ensures people to be treated equally as "voters." Suppose that two people, say, j, k, exchange their nominations each other. Anonymity says that this exchange should not affect the winning of any other person, i, so that j and k have the same influence on i's winning.

[1] Positive unanimity says that a person should be the (unique) winner if she is nominated by everyone else. Negative unanimity says that a person should not win if she is not nominated by anybody.