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Workflows for generating tetrahedral meshes for finite element simulations on complex geological structures

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ABSTRACT

Subsurface processing numerical simulations require accurate discretization of the modeling domain such that the geological units are represented correctly. Unstructured tetrahedral grids are particularly flexible in adapting to the shape of geo-bodies and are used in many finite element codes. In order to generate a tetrahedral mesh on a 3D geological model, the tetrahedrons have to belong completely to one geological unit and have to describe geological boundaries by connected facets of tetrahedrons. This is especially complicated at the contact points between several units and for irregular sharp-shaped bodies, especially in case of faulted zones. This study develops, tests and validates three workflows to generate a good tetrahedral mesh from a geological basis model. The tessellation of the model needs (i) to be of good quality to guarantee a stable calculation, (ii) to include certain nodes to apply boundary conditions for the numerical solution, and (iii) support local mesh refinement. As a test case we use the simulation of a transient electromagnetic measurement above a salt diapir. We can show that the suggested workflows lead to a tessellation of the structure on which the simulation can be run robustly. All workflows show advantages and disadvantages with respect to the workload, the control the user has over the resulting mesh and the skills in software handling that are required.

1. Introduction

Modern methods for exploring mineral and energy resources and new techniques for underground monitoring rely on numerical simulations of physical state variables like stress field, heat flux or geothermal potential. Recent studies include simulations of fluid flow in oil or groundwater reservoirs (Oladyshkin et al., 2007; Park et al., 2014; Wycisk et al., 2009), the migration of CO_2 at a sequestration site (Kempka et al., 2013), the temperature and heat flow for planning heat exchanger systems (Kohl et al., 2003), stress and strain fields for localizing potential flank instabilities in mountains (Apuani et al., 2013), seismic wavefields for optimizing measurement parameters (Wang et al., 2011; Lambert et al., 2013), and simulations of electromagnetic fields (Rücker et al., 2006; Schwarzbach et al., 2011).

The simulation results obtained are only meaningful if the geology of the studied region is represented correctly. Therefore, successful exploration or monitoring requires a realistic 3D model of the underground geometry, integrating all available geodata. This has to be constructed in special geomodeling software adapted to the peculiarities of geological data like scarcity and uncertainty. Subsequently, other specialized software has to be used for simulating the physical processes. This is generally done using, e.g., the finite element (FE), finite volume or the finite difference (FD) methods. Therefore, a general workflow is needed in order to perform a numerical simulation on a realistic model (Fig. 1).

Initial geological data are often represented by points indicating the position of a geological interface in a borehole and lines (points connected by segments) representing interfaces on a seismic section or geological map. Geological discontinuities like horizons or faults are modeled using the correlation and interpolation of data, while respecting constraints and geological concepts (Caumon, 2010).

Geological models describe the geometry of the subsurface either by a boundary representation or by discrete cells. Boundary representations describe the spatial extent of a geo-object only by its boundaries (Weiler, 1988; Mallet, 1989b; Duvinage et al., 1999). A coherent boundary representation is achieved when the volume of the body is completely confined and partitioned by surfaces without holes and overlaps. The surfaces can either be described by a discrete data model like a triangulated surface or by a function like a spline (Mallet, 2002).

In a cell representation, the modeling volume is discretized with cells, each of which belongs to one geological unit. To make a

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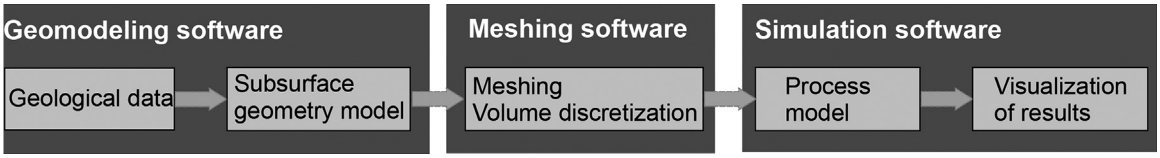

Fig. 1. General workflow for running a FE simulation on a 3D subsurface model.

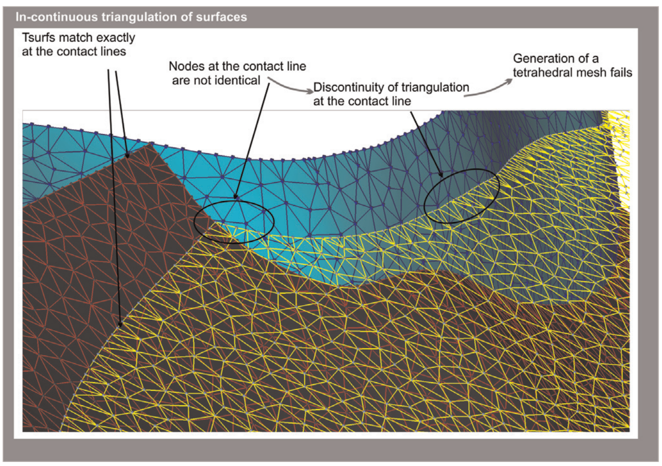

Fig. 2. Boundary representation of a geological body as produced by Gocad and Skua. The triangle nodes at the surface contacts are not identical; the resulting mesh is not continuous.

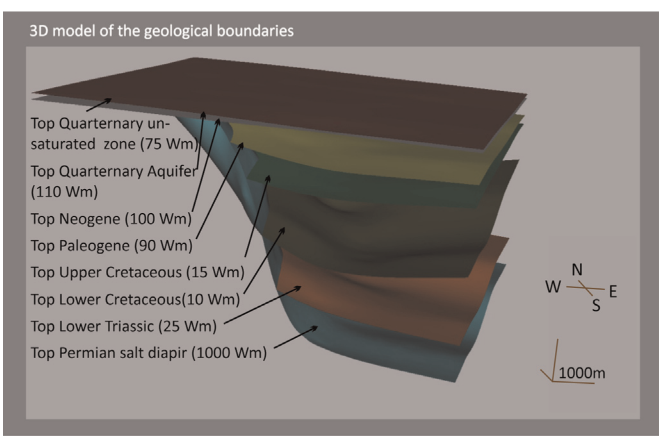

Fig. 3. 3D geometric model of the modeling area in Niedersachsen (Germany). Geological bodies with realistic geometries have to be transformed into a tetrahedral mesh. Each geological unit is characterized by a constant specific electrical resistivity.

Table 1
Formation resistivities for the test case "Salt diapir in Niedersachsen".

Formation	Resistivity ρ (Ωm)
Quaternary	90
Aquifer	110
Upper tertiary	100
Lower tertiary	60
Upper cretaceous	15
Lower cretaceous	12
Jurassic	10
Triassic	25
Zechstein salt	1000

process simulation, a cell representation of the modelling domain is needed. Depending on the method applied to solve the process equation numerically, different cell types are used. Structured grids consist of hexahedral cells characterized by a regular topology. They are often stored in a raster format and are represented by the number of cells and the step width in each dimension. The simplest type, the rectangular hexahedral grid, is used by FD codes in hydrogeology, such as Modflow (USGS) or Shemat (Clauser, 2003), or for seismic and resistivity modeling (Spitzer, 1995; Bohlen, 2002; Streich, 2009). This grid type approximates geological discontinuities by staircase-like boundaries (Fetel, 2006; Bistacchi et al., 2007).

Furthermore hexahedral cells can be deformed in order to follow the horizon geometry (Bennis et al., 1996) and this form can be used for FD simulations (Chambers et al., 1999; Lee et al., 2002; Kempka et al., 2013). One example is Schlumberger's reservoir simulation software Eclipse. However, Structured Grids (also called SGrids) become disconnected across faults, which is a problem for many FD codes.

Unstructured grids consist of an irregular pattern of grid points with neither a pre-defined topology nor fixed cell geometry. Unstructured cells can be described by a vector format, giving the corner nodes of each cell, and by a topological model, describing the neighborhood relations of the cells. The simplest type of unstructured grid is a tetrahedral mesh. It can adapt to complicated geometries (Moyen, 2005; Muron, 2005), since the tetrahedrons can represent sharp forms of geo-bodies. Each tetrahedron belongs to one geological unit only, and geological surfaces coincide with connected facets of the tetrahedral mesh. Unstructured tetrahedral meshes can be refined locally in order to achieve a good fit (Frank, 2006), are able to describe even the most complex structural situations and are often used with FE based software codes (Mallet, 2008).

In this study, we want to use a real-world geological model for the FE simulation of a transient electromagnetic field, which could be used to optimize the monitoring design for the leakage of dissolved carbon dioxide into shallow aquifers (Börner et al., 2013). The simulation code used (Afanasjew et al., 2013) works on tetrahedral meshes.

However, the development of workflows allowing the geoscientist to construct an unstructured mesh suitable for FE simulations hitherto lagged behind the different options for constructing SGrids in commercial modeling packages. A workflow for the generation of unstructured tetrahedral meshes on a complex geological model has to fulfill the following requirements:

1. *Sufficient mesh quality for running a process simulation.* The tetrahedrons should not be too acute-angled, because numerical instabilities can occur. Therefore, the shape of the tetrahedrons has to be checked and if necessary improved.
2. *Incorporation of geometry for defining boundary conditions and constraints.* The formulation of boundary conditions for the numerical simulation often requires certain points or lines to be part of the mesh. The workflow should be able to add these objects as constraints to the tessellation.
3. *Local adaption.* Local refinement of the mesh has to be possible. In our example, the mesh has to be refined in the vicinity of physical sources and receivers to avoid numerical errors during the simulation.