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On budget balance of the dynamic pivot mechanism

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ABSTRACT

We modify the dynamic pivot mechanism of Bergemann and Välimäki (*Econometrica*, 2010) in such a way that lump-sum fees are collected from the players. We show that the modified mechanism satisfies ex-ante budget balance as well as ex-post efficiency, periodic ex-post incentive compatibility, and periodic ex-post individual rationality, as long as the Markov chain representing the evolution of players' private information is irreducible and aperiodic and players are sufficiently patient. We also show that the diverse preference assumption of Bergemann and Välimäki may preclude budget balance.

1. Introduction

The research on dynamic mechanism design is recently surging. As the surveys of Bergemann and Said (2010) and Vohra (2012) suggest, the literature can be divided into two categories. The first category deals with the environments in which the population of players changes over time, but their private information is fixed, whereas the second category deals with the environments in which the population of players is fixed, but their private information changes over time.

This paper belongs to the second category. Notable papers in this category include Athey and Segal (2013) and Pavan, Segal, and Toikka (2014). In particular, Bergemann and Välimäki (2010) introduced the dynamic pivot mechanism, which is a generalization of the renowned VCG (Vickrey–Clarke–Groves) mechanism to the dynamic setting. The dynamic pivot mechanism is ex-post efficient, periodic ex-post incentive compatible, and periodic ex-post individually rational. The problem with this mechanism, however, is that it may run an expected budget deficit. In this paper, we modify the dynamic pivot mechanism in a way that lump-sum fees are collected from the players so that the modified mechanism satisfies ex-ante budget balance as well as the aforementioned property of ex-post efficiency, periodic ex-post incentive compatibility, and periodic ex-post individual rationality.

In the next section, we set up a general model and establish that the dynamic pivot mechanism with lump-sum fees is ex-post efficient, periodic ex-post incentive compatible and individually rational, and ex-ante budget balancing if the Markov chain representing the evolution of players' private information is irreducible and aperiodic and players are sufficiently patient. This result holds essentially since the participation constraints of the worst-off types are relaxed in the

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dynamic setting. On the other hand, we show that the diverse preference assumption of Bergemann and Välimäki (2010) may preclude budget balance. The reason is that this assumption makes the Markov chain reducible. Section 3 contains discussion on recent related research and future agenda.

2. Main results

We first present a general model based on Bergemann and Välimäki (2010), and then discuss budget balance of the dynamic pivot mechanism.

2.1. The setup

There is a set $I = \{1, \ldots, n\}$ of players and a countable number of periods, indexed by $t \in \{0, 1, \ldots\}$. Player i's type in period t is $\theta_i^t \in \Theta_i$. We assume that this is private information. Let $\theta^t = (\theta_1^t, \ldots, \theta_n^t)$ and $\Theta = \prod_{i=1}^n \Theta_i$.[2] To deliver the main idea without getting involved into many subtle mathematical issues, we will assume that Θ is *discrete* (i.e., finite or countably infinite). After $\theta^t \in \Theta$ is realized in period t, a public action $a^t \in A$ is determined. In addition, let $z_i^t \in \mathbb{R}$ be a monetary transfer *from* player i in period t. Given sequences $(\theta^0, \theta^1, \ldots)$ of type profiles and $(a^0, a^1, \ldots)$ of actions, together with $(z_i^0, z_i^1, \ldots)$ of i's monetary transfers, player i's payoff is

$$\sum_{t=0}^{\infty} \delta^t \Big(v_i(\theta_i^t, a^t) - z_i^t \Big),$$

where (i) δ is a common discount factor and $\delta < 1$, and (ii) $v_i(\cdot)$ is a reward function. Note that we deal with the private-values environment in that player i's reward function depends only on player i's type. We also assume that $|v_i(\theta_i, a)| \leq C < \infty$ for all i, θ_i, and a.

The dynamic evolution of players' types is represented by a Markov chain. Let $p(\theta^{t+1}|\theta^t, a^t)$ be the transition probability that type profile θ^{t+1} will be realized in period $t+1$ when the type profile is θ^t and the action is a^t in period t. Observe that, except for the fact that θ is private information, this environment fits into a Markov decision process with Θ being the set of states.[3]

We consider a dynamic direct mechanism that asks each player to report her type (i.e., state) in each period. Let r_i^t denote player i's report in period t, which may or may not be equal to her true type θ_i^t. Let

$$h_i^t = (\theta_i^0, r^0, a^0, \theta_i^1, r^1, a^1, \ldots, \theta_i^{t-1}, r^{t-1}, a^{t-1}, \theta_i^t)$$

be a private history of player i in period t, where each $r^s = (r_1^s, \ldots, r_n^s)$ for $s = 0, \ldots, t-1$ is a report profile, and let H_i^t be the set of all such histories. A (pure) strategy for player i in period t is a function $\hat{r}_i^t : H_i^t \to \Theta_i$. A strategy is truth-telling if $\hat{r}_i^t(h_i^t) = \theta_i^t$ for all h_i^t. In addition, let

$$h^t = (r^0, a^0, r^1, a^1, \ldots, r^{t-1}, a^{t-1}, r^t)$$

be a public history in period t and let H^t be the set of all such histories. Observe that, when players adopt the truth-telling strategy, the private histories do not contain more information than the public histories on the equilibrium path. Since we are mainly concerned with incentive compatible mechanisms in which the truth-telling strategy is an equilibrium, we will not henceforth distinguish between true states and reported states (mainly to save notations).

In each period, the mechanism decides on the action based on the actions chosen up to the previous period and the reports up to the beginning of this period. Thus, when players adopt the truth-telling strategy, a deterministic (history-dependent) decision rule of the mechanism in period t is a function $\hat{a}^t : H^t \to A$. A special class of decision rule is the deterministic Markovian decision rule that chooses an action based only on the current state, i.e., $\hat{a}^t : \Theta \to A$. Moreover, a randomized decision rule $\hat{a}^t$ specifies a probability distribution on the set of actions. Randomized decision rules may be history-dependent or Markovian. A *policy* of the mechanism is a sequence of decision rules $\pi = (\hat{a}^0, \hat{a}^1, \ldots)$. Let Π be the set of all policies.

An efficient policy is $\pi^* \in \Pi$ that maximizes the expected discounted sum of players' rewards. That is,

$$\pi^* \in \arg\max_{\pi \in \Pi} \; E_\theta^\pi \Big[\sum_{t=0}^{\infty} \delta^t \sum_{i=1}^{n} v_i(\tilde{\theta}_i^t, \tilde{a}^t) \Big]$$

for every $\theta \in \Theta$. By the way, we will assume throughout that the relevant maximum is attained without specifying sufficient conditions. Since Θ is discrete, this assumption is valid when (i) A is finite or (ii) A is compact, $v_i(\theta_i, a)$ is continuous in

[2] We may include public information, say $\theta_0^t \in \Theta_0$, to be more realistic. We dispense with this additional notation for clearer presentation of the main idea.

[3] Note that this Markov formulation is essentially without loss of generality, since any dynamic model can be described using Markov notation by expanding the state space appropriately. See Stokey and Lucas (1989) or Athey and Segal (2013) for an excellent discussion.

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