Games and Economic Behavior 94 (2015) 214–221

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Note

Existence of monotone equilibrium in first price auctions with private risk aversion and private initial wealth ☆

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ARTICLE INFO

Article history:
Received 12 January 2015
Available online 28 July 2015

JEL classification:
C7
D7

Keywords:
Constant absolute risk aversion (CARA)
Constant relative risk aversion (CRRA)
First price auction
Initial wealth
Monotone equilibrium

ABSTRACT

In this paper, we study the existence of monotone equilibrium in first price auctions where bidders have a three-dimensional private type, i.e. their private values, degrees of risk aversion and initial wealth. Bidders' utility functions belong to the class of constant relative risk aversion (CRRA) or constant absolute risk aversion (CARA). The bidders' types are independent across bidders, while a bidder's private value, initial wealth and degree of risk aversion are allowed to be correlated. We show that a monotone equilibrium always exists in a general setting allowing for asymmetric bidders. Moreover, with symmetric bidders, a symmetric monotone equilibrium strategy must exist. A bidder's equilibrium strategy increases with bidders' private values and degrees of risk aversion. When bidders have CRRA utility, equilibrium bids decrease with initial wealth; when bidders have CARA utility, equilibrium bids are invariant to initial wealth.

1. Introduction

Risk aversion is a core notion for analyzing economic agents' decisions under uncertainty. Since Pratt (1964)'s formalization of risk aversion, a rich literature has been devoted to analyzing behavior of risk averse agents in a variety of situations. In particular, a large body of theoretical, experimental and empirical research has demonstrated that bidders' risk aversion is an important determinant of their bidding behavior in auctions. As is well known, risk aversion leads bidders to bid more aggressively in first-price auctions relative to other standard formats, helping to rationalize the extensively observed "overbidding" relative to the risk-neutral Bayesian Nash equilibrium in experiments (e.g. Cox et al., 1988 and Goeree et al., 2002 among others). Adopting a structural approach, Bajari and Hortaçsu (2005) show that the risk aversion model provides the best fit to experimental data among several competing models. Abundant evidence for bidders' risk aversion has also been identified in field auction data, including Baldwin (1995), Athey and Levin (2001), Perrigne (2003), Lu and Perrigne (2008), Campo et al. (2011), Campo (2012), Fang and Tang (2014), and Li et al. (2015) among others.

Two commonly adopted measures of risk aversion are the Arrow–Pratt coefficient of absolute risk aversion and the Arrow–Pratt–De Finetti coefficient of relative risk aversion. A Bernoulli utility function that generates a constant coefficient

☆ We are grateful to the editors and two anonymous reviewers for very insightful comments and suggestions that significantly improved the quality of the paper. We thank Ali Khan, Bin Liu and Yeneng Sun for their very helpful discussions. Lu gratefully acknowledges the financial support from Singapore Ministry of Education Academic Research Fund Tier 1. All remaining errors are our own.

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http://dx.doi.org/10.1016/j.geb.2015.07.003
0899-8256/

of absolute risk aversion is called a constant absolute risk aversion (CARA) utility function, while a utility function that generates a constant coefficient of relative risk aversion is called a constant relative risk aversion (CRRA) utility function. These two classes of utility functions are the preeminent tools by which to model risk aversion in a wide range of settings, including auctions in particular.

It has long been recognized in the auction literature that bidders' risk preferences can be heterogeneous and that such heterogeneous risk preferences can be bidders' private information. Cox et al. (1982a, 1982b) provide the first characterizations of closed form pure strategy equilibria in first-price auctions when bidders' degrees of risk aversion are modeled as part of their private types. Specifically, they adopt a constant relative risk aversion (CRRA) framework and assume the bidders' private values follow a uniform distribution. Cox et al. (1988) further find that a CRRA model with private degrees of risk aversion fits well the "overbidding" data generated by their first price auction experiments. In these studies, however, the existence of a pure strategy equilibrium is not fully established, since the hypothetical bidding strategy is only partially identified and it is thus difficult to verify its monotonicity over the whole ranges of degrees of risk aversion and values. Van Boening et al. (1998) further explore numerically solving the differential equations that a hypothetical equilibrium bidding strategy must satisfy. Their simulation results verify the existence of isotone equilibrium for a specific CRRA model with four bidders whose values follow a uniform distribution and their degrees of risk aversion follow a beta distribution on $[0, 1]$. The bidding strategies obtained increase in both bidders' degrees of risk aversion and their values. In all these studies, the initial wealth of bidders is fixed at zero.

Assuming the existence of pure strategy equilibrium in the model of Cox et al. (1988) and Van Boening et al. (1998), Pevnitskaya (2001) extends Levin and Smith (1994) by allowing for bidders who hold private information on their degrees of risk aversion before making their entry decisions. Potential bidders who observe their private degrees of risk aversion must decide simultaneously whether to incur an entry cost to discover their values before bidding in a first price auction. Pevnitskaya (2001) finds a self-selection effect that relatively more risk tolerant bidders choose to enter in a symmetric entry equilibrium, and this effect increases with the entry cost. This self-selection effect leads to less aggressive bids than those from an exogenous pool of bidders. The experimental study conducted by Palfrey and Pevnitskaya (2008) confirms these theoretical predictions.

Nevertheless, even with fixed initial wealth for bidders, the existence of monotone equilibrium in first price auction remains an open question in a general CRRA model with an arbitrary set of bidders whose joint distributions of values and degrees of risk aversion are left unrestricted. Verifying the existence of monotone equilibrium for a general model by checking the properties of the solution of the differential equations that Cox et al. (1988) and Van Boening et al. (1998) identify is not easy to implement. It is in general infeasible to study the properties of the hypothetical strategy for a general model without solving these equations analytically. If instead the bidders' utility functions are assumed to have a CARA form, this procedure would face new challenge since the linear segment of the bidding strategy identified in the existing literature on the CRRA case no longer holds. More technical complications could arise if correlation between bidders' value and degree of risk aversion is introduced into the CRRA or CARA model.

While values and private risk preferences have received relatively more attention in the literature, in settings with risk aversion it is also natural to interpret bidders' initial wealth levels as being private information. With risk averse bidders, variation in initial wealth would in general affect bidding behavior, but little is known about properties of equilibrium when wealth is private. In this study, we establish the existence of isotone equilibrium in first price auctions when bidders have three dimensional private types, i.e. their initial wealth levels, private values and degrees of risk aversion. To our knowledge, existence in this setting has not been previously studied in general.

To avoid the difficulties of solving differential equations that a hypothetical equilibrium strategy must satisfy, we follow an alternative approach along the lines of Reny (1999, 2011), Athey (2001) and McAdams (2003) to address the existence of monotone equilibrium in first price auctions where bidders have three-dimensional private information about their values, initial wealth and degrees of risk aversion. Our approach accommodates both CRRA and CARA specifications with an arbitrary number of potentially asymmetric bidders. We assume that private information—i.e. bidders' values, degree of risk aversion, and levels of initial wealth—is drawn independently across bidders, but leaves the joint distribution of values, initial wealth, and risk aversion for each bidder unrestricted. We find that a pure strategy equilibrium that is monotone in all three dimensions of private information must exist. Moreover, a symmetric pure strategy monotone equilibrium must exist for symmetric bidders. Our results thus complement existing studies on private risk aversion and strengthen the foundation for future theoretical, experimental and empirical studies on first price auctions when bidders are endowed with private information on their risk attitudes and/or initial wealth.

The rest of the paper is organized as follows. Section 2 describes the model and outlines notation used to unify discussion of the CARA and CRRA cases. Section 3 establishes existence of monotone equilibria in the general model with asymmetric bidders. Section 4 specializes this to existence of symmetric monotone equilibria in settings with symmetric bidders. Section 5 concludes. Some technical proofs are relegated to the online appendix.

2. First price auctions with private values, wealth and risk aversion

In this paper, we consider a sealed-bid first price auction of a single item. A reservation price r is imposed. Ties in winning bids are broken randomly. There are N potentially asymmetric bidders who are risk averse. Each bidder is endowed with three-dimensional private information about his value, initial wealth and degree of risk aversion.