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Maximizing social welfare in congestion games via redistribution

Victor Naroditskiy [a,*], Richard Steinberg [b]

[a] University of Southampton, United Kingdom
[b] London School of Economics and Political Science, United Kingdom

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ABSTRACT

It is well known that efficient use of congestible resources can be achieved via marginal pricing; however, payments collected from the agents generate a budget surplus, which reduces social welfare. We show that an asymptotically first-best solution in the number of agents can be achieved by the appropriate redistribution of the budget surplus back to the agents.

1. Introduction

Congestion games model situations in which multiple agents use shared resources, where each agent's value of a resource decreases with the total usage of the resource or, equivalently, the corresponding level of congestion, a negative externality. Thus, the higher the level of congestion, the less valuable the resource is to an agent. From the viewpoint of social welfare, the best use of resources occurs when the sum of the agents' values is maximized, i.e., when the use of resources is efficient.

In order to direct self-interested agents towards the efficient use of resources, one can resort to pricing. Specifically, requiring each agent to pay for the corresponding disutility she imposes on others[1] results in an efficient use of the resources (see, e.g., MacKie-Mason and Varian, 1995; Kelly, 1997). While such pricing will maximize the agents' total value, each agent who makes a payment will suffer a reduction in utility. In some contexts, the collected revenue is desirable, as it increases the utility of the party collecting it (e.g., the seller). However, in many congestion scenarios the main objective is the welfare of the agents, which is decreased by any payments collected (Cole et al., 2006). Indeed, congestion scenarios often arise in settings where the resources are intended for public use and not for the generation of revenue. In this work we ask: How can social welfare be maximized in congestion games? More specifically: *How can most of the revenue be redistributed back to the agents while ensuring that an efficient allocation is achieved?*

In the first part of the paper, after observing that an atomic congestion game can be modeled as the allocation of multiple copies of heterogeneous items, we show that as the number of agents increases, all of the revenue can be redistributed asymptotically while still achieving the efficient allocation. Specifically, we prove that a redistribution rule designed for non-congestion models by Bailey (1997) and generalized by Cavallo (2006) asymptotically achieves full budget balance

* Corresponding author.
E-mail address: vnarodit@gmail.com (V. Naroditskiy).

[1] These payments are known as *Pigouvian taxes* (Pigou, 1920), *marginal cost prices*, or *congestion prices*.

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in the presence of congestion. Thus, we identify a first-best solution to the problem of welfare maximization in atomic congestion games. It is interesting to observe that while in non-congestion settings (e.g., allocating multiple copies for an identical item to agents with unit demand) the revenue redistributed by Bailey–Cavallo can be arbitrarily low, we show here that in congestion settings (asymptotically) all of the revenue is redistributed.

In the atomic context, we also clarify the relationship between congestion prices and VCG payments by proving that, as the efficient level of congestion increases, the two asymptotically approach each other. Intuitively, both congestion prices and VCG payments charge the agent for the "externality" she imposes on others, and therefore, the connection between them is not entirely surprising. However, we have not seen a formal analysis of the relationship between the prices, while the two are normally treated separately (see, e.g., Sections 9.1 and 9.5 in Courcoubetis and Weber, 2003).

In the last section of the paper prior to the Discussion, we turn to non-atomic congestion games. We show that revenue can be redistributed to the agents in equal shares, resulting in a first-best solution. As the effect of an individual agent on the total congestion in an atomic congestion game becomes negligibly small, the redistributed amounts for atomic and nonatomic models coincide.

We now place our work within the existing literature. Revenue redistribution has received considerable attention in the work on mechanism design, mostly in the context of dominant-strategy implementation in allocation models. Specifically, the agenda of welfare maximization in models without a residual claimant has been pursued in the allocation of identical items (Moulin, 2009; Guo and Conitzer, 2008, 2009; de Clippel et al., 2014), the allocation of non-identical items (Guo, 2012), in public good settings (Naroditskiy et al., 2012), and for the general application of redistribution (Cavallo, 2006). However, the revenue redistribution literature has thus far not considered scenarios with congestion, even though redistribution is important there. Indeed, many congestion scenarios are characterized by the lack of a residual claimant (as argued above), and high amounts of revenue collected before redistribution (as argued below). In this paper, we apply the rule from (Cavallo, 2006) to redistribute revenue in a model with congestion.

Following this revenue redistribution literature, our results on atomic congestion games are derived in the context of centralized mechanisms. Congestion games are usually considered in a decentralized context. However, centralized mechanisms have been applied to the study of congestion games, e.g., in the context of computational complexity (Chakrabarty et al., 2005; Blumrosen and Dobzinski, 2007). In more detail, Blumrosen and Dobzinski (2007) derive computational complexity of finding welfare-maximizing use of resources. The model they adopt is also different. In our atomic case, agents have combinatorial preferences over the resources but share the same congestion function. In their case, congestion functions are player specific, and agents have non-combinatorial valuations over resources.

Well-studied in the field of congestion pricing are routing and traffic equilibria models (see, e.g., Roughgarden, 2005). In these models agents are to be routed along the edges of a network with each agent associated with a sink node and a source node. Edges on the network differ in their capacity to carry traffic as reflected by edge-specific *congestion functions*. A congestion function specifies the cost each agent routed along the edge experiences. The objective is to induce a congestion-minimizing flow when agents make their routing choices selfishly. However, the agents may arrive at a non-efficient Nash equilibrium. A seminal text (Beckmann et al., 1956) shows how edges can be priced in order to induce the efficient flow in Nash equilibrium. Specifically, each edge is associated with the *congestion price* equal to the marginal cost incurred by the agents using the edge.

The question of welfare-maximization in congestion domains has been considered before. Cole et al. (2006) investigated how congestion prices can be modified when no redistribution is possible. Redistribution of revenue was studied by Adler and Cetin (2001) within a model that had been introduced by Vickrey (1969). In Vickrey's model, congestion is manifested in waiting times to use a resource. Agents form a queue, and the waiting time of each is given by their position in the queue. Prices can be used to eliminate waiting times by encouraging each agent to deviate from her preferred departure time. Thus, agents have different costs. Deviation from the ideal departure time can be interpreted as the level of service provided—the greater the deviation, the lower the level of service. Payments of agents receiving a higher level of service can be redistributed to agents receiving a lower level of service. In contrast, in this paper we focus on a model where each allocated agent obtains the same level of service, i.e., experiences the same level of congestion, which is the case in congestion games.

The remainder of the paper is structured as follows. In Section 2 we show that a first-best solution to welfare-maximization in atomic congestion games is provided by applying the Bailey–Cavallo redistribution rule. We derive the result for the single-resource congestion game first, and then for general congestion games. This section also shows that congestion prices and VCG payments in congestion settings are fundamentally similar. Section 3 talks about nonatomic routing games. There we prove that revenue can be redistributed in equal shares. Concluding remarks appear in Section 4.

2. Centralized solution: atomic congestion games

This section focuses on games where the number of players is finite and each may have a non-negligible effect on congestion. We describe how congestion effects from these games can be represented in an allocation model, and apply a redistribution rule that was designed for allocation settings.

The mapping between congestion and allocation models will make it clear that the impossibility result regarding fully budget-balanced efficient mechanisms (Green and Laffont, 1977; Holmstrom, 1979) applies to our model.