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Agreeing to agree and Dutch books ☆

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ABSTRACT

We say that agreeing to agree is possible for an event E if there exist posterior beliefs of the agents with a common prior such that it is common knowledge that the agents' posteriors for E coincide. We propose a notion called Dutch book which is a profile of interim contracts between an outsider and the agents based on the occurrence of E, such that the outsider makes positive profit in all states. We show that in a finite state space, when the agents cannot tell whether E occurred or not, agreeing to agree is possible for E if and only if there is no Dutch book on E. This characterization also holds in countable state spaces with two agents. We weaken the notion of Dutch book to characterize agreeing to agree in a countable state space with multiple agents, when each set in each agent's information partition is finite.

1. Introduction

Aumann (1976) demonstrates the impossibility of agreeing to disagree: for any posteriors with a common prior, if the agents' posteriors for an event E are different (they disagree), then the agents cannot have common knowledge (agreement) of these posteriors. Here we ask what are the properties of an event E that make agreeing to *agree* possible. That is, under what conditions there are posteriors of the agents that are derived from a common prior, such that the agents have common knowledge (agreement) that their posteriors of E are the same (agree). Lehrer and Samet (2011) obtained a characterization of the possibility of agreeing to agree for two agents. In this paper, we study the possibility of agreeing to agree for an arbitrary finite set of agents.

Clearly, in any state at which an agent's posterior for E is nontrivial, the agent cannot tell whether E occurred or not. We say in this case that the agent is ignorant of E. Lehrer and Samet (2011) observed that for two agents in a finite state space, ignorance of all agents in all states is also a sufficient condition for agreeing to agree. Consider the following example from Lehrer and Samet (2011).

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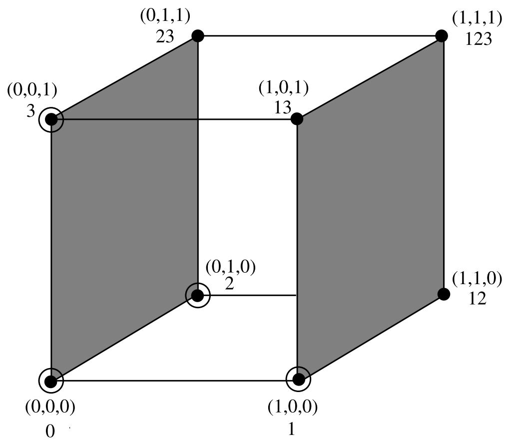

The eight states in Example 2 are depicted as black dots. The partition of firm 1 consists of two sets of states marked by the dark faces of the cube. The partitions of 2 and 3 are similarly defined. The four states of event E are circled. Firm 1 is ignorant of E: in each of its partition elements, it does not know whether E or $\neg E$ occurred. Similarly, the other two firms are ignorant of E. Yet, there is no common prior such that the posterior probabilities of E are the same for all firms in all states.

Fig. 1. Ignorance does not imply possibility of agreeing to agree.

Example 1. Consider two firms 1, 2. Firm i can be either profitable, denoted by $x_i = 1$, or unprofitable, $x_i = 0$. There are four possible states of the world of the form (x_1, x_2), where each x_i is 0 or 1. Firm i knows only whether it is profitable or not. We denote by 0 the state $(0, 0)$ and all other states are denoted by the names of the profitable firms.

Consider first the event E that both firms have the same financial situation. That is, E consists of the state where $x_1 = x_2 = 0$ and $x_1 = x_2 = 1$. Clearly, both agents are ignorant of E at every state. There are many posteriors that have a common prior such that the firms have common knowledge that the posterior probabilities of E coincide. For instance, if we take the uniform distribution on the four states as the common prior, then both firms have the same posterior for E namely, $1/2$.

However, the characterization of Lehrer and Samet (2011) does not hold when the number of agents exceeds two. Specifically, ignorance is no longer sufficient for agreeing to agree with more than two agents, as demonstrated by the following example.

Example 2. Consider three firms 1, 2, 3. Firm i can be either profitable, denoted by $x_i = 1$, or unprofitable, $x_i = 0$. There are eight possible states of the world of the form (x_1, x_2, x_3), where each x_i is 0 or 1. Firm i knows only whether it is profitable or not. We denote by 0 the state $(0, 0, 0)$ and all other states are denoted by the names of the profitable firms. For example, the state $(1, 0, 1)$ is denoted by 13.

The states of the world are the vertices of the unit cube in $\mathbb{R}^3$. The partition for firm i consists of the face of the cube where $x_i = 0$ and the face where $x_i = 1$ (see Fig. 1).

Let E be the event that no more than one firm is profitable. That is, $E = \{0, 1, 2, 3\}$. Since each face contains at least one point from E and one point from $\neg E$, each firm is ignorant of E.

Suppose that agreeing to agree is possible for E, with posteriors of E being constantly p, and P is a common prior of the types. Consider the face $x_1 = 1$. If the probability of this face is positive, then the requirement that the posterior of E in this face is p says that $P(1)/p = P(12, 13, 123)/(1 - p)$. If the probability of the face is 0, then this equality trivially holds. Writing the similar equations for the faces $x_2 = 1$ and $x_3 = 1$ and summing the equations, we have: $P(1, 2, 3)/p = [2P(12, 13, 23, 123) + P(123)]/(1 - p)$. The right hand side is at least $2P(\neg E)/(1 - p) = 2$ and the left hand side is at most $P(E)/p = 1$.[1] We reach a contradiction. Thus, agreeing to agree is impossible although all firms are ignorant in every state.

This example calls for an alternative characterization of agreeing to agree. For this purpose, we introduce the notion of Dutch book. Consider the following contract between an outsider and some agent i. The contract specifies an amount f_i to be transferred from i to the outsider if E is the case and in the opposite direction if E is not the case. The transfer is made ex post, that is, it requires the knowledge of the state, or at least the knowledge whether E is true or not true in the state. However, the contract is an interim contract, in the sense that the amount transferred, f_i, is known to i.[2] A *Dutch book* is a profile of such interim contracts, one for each agent, under which the outsider profits regardless of the true state.[3] Note

[1] Since the posteriors of E are all p, it follows that $P(E) = p$ and thus $P(\neg E) = 1 - p$.

[2] That is, $f_i : \Omega \to \mathbb{R}$ is measurable with respect to the information partition of each agent; see Section 3.1.

[3] Thanks to ignorance, posterior probabilities exist such that by accepting a Dutch book, each agent obtains a positive expected payoff in every states.