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Tie-breaks and bid-caps in all-pay auctions ☆

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ABSTRACT

We revisit the two bidder complete information all-pay auction with bid-caps introduced by Che and Gale (1998), dropping their assumption that tie-breaking must be symmetric. Any choice of tie-breaking rule leads to a different set of Nash equilibria. Compared to the optimal bid-cap of Che and Gale we obtain that in order to maximize the sum of bids, the designer prefers to set a less restrictive bid-cap combined with a tie-breaking rule which slightly favors the weaker bidder. Moreover, the designer is better off breaking ties deterministically in favor of the weak bidder than symmetrically.

1. Introduction

At the Olympic Games of 1896, the first Olympic Games of the Modern Era, two weightlifters impressed the Athenian audience particularly: the Scotsman Launceston Elliot and the Dane Viggo Jensen. Both lifted the same highest weight. The jury decided to solve the tie in favor of Jensen for he was considered to have the better style. Unfamiliar with this tie-breaking rule, the British delegates protested against the decision. This finally led to Elliot and Jensen obtaining the permission to try again for lifting higher weights. Yet both failed. In the end, Jensen was declared the champion.

Nowadays, it is neither style nor the energy of the own country's delegates that helps to win a tie in a weightlifting-contest. Instead, when a tie occurs, it is resolved in favor of the lighter athlete. Behind this is the idea that a lighter athlete, though in the same weight-class, probably has to exert more effort to lift the same weight than a heavier competitor.

Clearly, sports contests are more interesting if athletes display great efforts. For a designer, it is hence a natural objective to maximize the sum of efforts exerted by the contestants. Che and Gale (1998) show that handicaps can be an effective tool for raising aggregate effort levels in all-pay contests but they restrict their analysis to symmetric tie-breaking. We are going to allow the designer not only to set handicaps optimally, but also to choose the optimal tie-breaking rule.

In a complete-information all-pay auction without bid caps, the stronger bidder's advantage arises from his ability to win with certainty. He can secure a positive payoff by bidding just above the weak bidder's valuation. If a bid cap less than the weak bidder's valuation is imposed, this advantage disappears. Moreover, the advantage can be reversed if the tie-breaking is sufficiently biased towards the weak bidder, since the weak bidder can then guarantee himself a positive payoff by bidding the cap. In fact, every choice of tie-breaking rule leads to a different set of Nash equilibria.

We provide a complete characterization of the rich equilibrium structure. By choosing an appropriate combination of tie-breaking rule and bid-cap, the designer can enforce pure equilibria as well as mixed equilibria in which either of the

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bidders earns zero payoff. Compared to Che and Gale's optimal bid-cap under symmetric tie-breaking, we obtain that the designer optimally sets a less restrictive bid-cap combined with a tie-breaking rule which slightly favors the ex ante weaker bidder. Whereas the unique Nash equilibrium of the unrestricted all-pay auction is in mixed strategies, both of these policies force bidders to play a pure strategy equilibrium. Both bidders bid the bid-cap. The optimal policy exploits the fact that the weaker bidder is willing to bid more if tie-breaking is biased in his favor. If this bias is not too large this does not deter the stronger bidder from competing.

In many real-world settings, ties are broken either in favor of one bidder or 50 : 50. Therefore, we also consider the designer's problem if he is restricted to choosing between symmetric and deterministic tie-breaking rules. Even under this restriction, the designer can do better than in the optimal policy of Che and Gale (1998). In the optimum, he sets a bid-cap which is just small enough to influence equilibrium behavior and always breaks ties in favor of the weak bidder. Superficially, this policy seems like a minimal intervention into the game but it has important consequences. In an unrestricted all-pay auction, there is a mixed equilibrium in which the weak bidder stays out with a positive probability. The designer's policy gives rise to an equilibrium in which the weak bidder makes a preemptive bid (by bidding at the bid-cap) with the same probability with which he would stay out in the unrestricted auction.

Aggregate bids vary across policies as soon as asymmetries are present. Differences in aggregate bids increase as asymmetries grow larger. In the limit, the policy of Che and Gale outperforms the standard unrestricted all-pay auction by a factor of two. Solving the tie in favor of the weak bidder combined with the optimal bid-cap leads to an increase by a factor of three. If a tie-breaking slightly in favor of the weak bidder is possible, the optimal policy outperforms the standard auction by a factor of four.

Related literature In the vast literature on all-pay auctions,[1] tie-breaking rules have received comparatively little attention. Indeed, in many all-pay auction games, the set of Nash equilibria is invariant to the choice of tie-breaking rule. An example is a standard complete information all-pay auction in which at least two bidders have positive valuations for the object for sale.[2] In other related games, the choice of tie-breaking rule is a necessity since a Nash equilibrium exists only for certain tie-breaking rules. Consider for instance a two-player complete information all-pay auctions in which bidders have valuations $v_1 > 0$ and $v_2 = 0$. Then a Nash equilibrium (in which both bidders bid zero) exists only if tie-breaking always favors bidder 1.

In contrast, in all-pay auctions with binding bid-caps the choice of tie-breaking rule is decisive since in equilibrium both bidders play the bid-cap with positive probability. Yet in the literature only the case of symmetric tie-breaking has been considered. This concerns both the complete information case studied by Che and Gale (1998), Persico and Sahuguet (2006)[3] and Hart (2014),[4] and the incomplete information case studied by Gavious et al. (2003) and Sahuguet (2006). See Che and Gale (1998) for a discussion of the relation to policies other than bid-caps such as minimum-bid requirements.

Outline The paper is structured as follows. Section 2 introduces the model. Section 3 characterizes the bidders' equilibrium behavior for all combinations of bid-caps and tie-breaking rules. Section 4 analyzes the designer's optimization problem. First we allow for arbitrary tie-breaking rules, then we focus on tie-breaking rules that are either deterministic or symmetric. Section 5 discusses some extensions and implications of our analysis. All proofs are in Appendix A.

2. The model

We consider a complete information all-pay auction with two bidders 1 and 2 with positive valuations v_1 and v_2 for winning. Throughout we assume $v_1 > v_2$. Each bidder is restricted to choose his bid b from the interval $[0, m]$ at a cost of b. If a bidder submits the strictly highest bid, he wins. If both bidders submit the same bid, bidder 1 wins with probability $\alpha \in [0, 1]$, otherwise bidder 2 wins. We assume that before the auction takes place a designer chooses α and the handicap-level m in order to maximize the sum of bids. This is the setting of Che and Gale (1998) with the only difference that they restrict their analysis to symmetric tie-breaking.

If the designer does not impose a bid-cap, i.e. $m = \infty$, we are back to the standard complete information all-pay auction. In its unique equilibrium, both bidders mix uniformly over $[0, v_2)$.[5] Moreover, bidder 2 bids 0 with probability $1 - \frac{v_2}{v_1}$. For $m > v_2$, this set of strategies remains the unique equilibrium. The case $m = 0$ is trivial. Thus we assume in the following that the designer chooses (α, m) from the set

$$C = [0, 1] \times (0, v_2].$$

We denote by C_G the subset analyzed by Che and Gale (1998), $C_G = \left\{\frac{1}{2}\right\} \times (0, v_2]$.

[1] See Konrad (2009) for an overview.

[2] This has been shown, among others and in increasing generality, by Hillman and Samet (1987), Hillman and Riley (1989), Baye et al. (1996), and Siegel (2009). A parallel result holds for the incomplete information case studied first by Weber (1985) and Hillman and Riley (1989).

[3] Persico and Sahuguet (2006) embed the model of Che and Gale (1998) into a model of electoral competition in which parties try to attract heterogeneous voters. In their setting, the symmetric tie-breaking is implemented via the assumption that undecided voters toss a fair coin.

[4] Hart (2014) departs from the symmetry in Che and Gale's setting via analyzing asymmetric bid-caps under symmetric tie-breaking. This is similar to considering a tie-breaking always in favor of the less restricted bidder and implementing the more rigid cap for both bidders.

[5] See, e.g., Hillman and Riley (1989).