ARTICLE IN PRESS
YGAME:2262

Games and Economic Behavior ••• (••••) •••–•••

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Limitations of randomized mechanisms for combinatorial auctions

Shaddin Dughmi[a,1], Jan Vondrák[b,*]

[a] University of Southern California, 941 Bloom Walk, Los Angeles, CA 90089, United States
[b] IBM Almaden Research Center, 650 Harry Rd, San Jose, CA, United States

ARTICLE INFO

Article history:
Received 1 March 2012
Available online xxxx

JEL classification:
C70
D44

Keywords:
Algorithmic mechanism design
Combinatorial auctions
Incentive compatibility
Submodular valuations

ABSTRACT

We address the following fundamental question in the area of incentive-compatible mechanism design: Are truthful-in-expectation mechanisms compatible with polynomial-time approximation? In particular, can polynomial-time truthful-in-expectation mechanisms achieve a near-optimal approximation ratio for combinatorial auctions with submodular valuations?

We prove that this is not the case: There is a constant $\gamma > 0$ such that there is no randomized mechanism that is truthful-in-expectation — or even approximately truthful-in-expectation — and guarantees an $m^{-\gamma}$-approximation to the optimal social welfare for combinatorial auctions with submodular valuations in the value oracle model. In contrast, a non-truthful $(1 - 1/e)$-approximation algorithm is known (Vondrák, 2008), and a truthful-in-expectation $(1 - 1/e)$-approximation mechanism was recently developed for the special case of coverage valuations (Dughmi et al., 2011b). We also prove an analogous result for the combinatorial public projects (CPP) problem. Both our results present a significant separation between coverage functions and submodular functions, which does not occur for these problems without strategic considerations.

1. Introduction

The overarching goal of *algorithmic mechanism design* is to design computationally-efficient mechanisms that solve or approximate fundamental resource allocation problems in which the underlying data is elicited from self-interested participants in the system. Work in this field has revealed a fundamental tension between the two main design goals in this domain, incentive-compatibility and computational efficiency. Understanding the power of mechanisms that satisfy both desiderata, in terms of their ability to approximate optimal allocations of resources, has therefore spawned a large literature of both positive and negative results that draw on ideas from algorithm design, computational complexity, and mechanism design.

In this paper, we consider mechanisms for *combinatorial auctions*. In combinatorial auctions, there is a set of items up for sale, and a set of self-interested players each of whom is equipped with a private *valuation function* mapping bundles of items to the player's value. The valuation functions are monotone non-decreasing, and normalized so that the value for the empty set is zero. We adopt the perspective of an auctioneer whose goal is to maximize social welfare, which in this context is the sum of the players' values for the bundles they receive. As is traditional in algorithmic mechanism design

* Corresponding author.
E-mail addresses: shaddin@usc.edu (S. Dughmi), jvondrak@us.ibm.com (J. Vondrák).
[1] This work was done while the author was a student at Stanford University.

0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2014.01.007

more generally, we examine the trade-off between incentive compatibility and computational efficiency[2] in this setting. Since we do not assume the existence of a prior on players' valuations, we focus on mechanisms that are dominant-strategy incentive compatible.

Combinatorial auctions enjoy paradigmatic status in mechanism design. We quote (Blumrosen and Nisan, 2010): "Combinatorial auctions serve as a common abstraction for many resource allocation problems in decentralized computerized systems such as the Internet, and may serve as a central building block of future electronic commerce systems." It is therefore unsurprising that combinatorial auctions have already been applied or considered in many contexts, such as in allocation of electromagnetic spectrum, allocation of airport take-off and landing slots, and more. (See Cramton et al. 2006.)

In many existing and potential applications of combinatorial auctions, the welfare-maximizing and incentive-compatible VCG mechanism cannot be deployed, due in part to the computational intractability of the welfare maximization problem. Combinatorial auctions that are actually employed, for instance in the practical settings mentioned above, are often heuristic in nature, specifically tailored to particular markets, and rigorous guarantees on their performance are rare in all but the simplest of settings. This has motivated a major research direction in algorithmic mechanism design, seeking a rigorous understanding of the space of computationally-efficient incentive-compatible mechanisms for variants of combinatorial auctions.

Strong impossibility results based on complexity-theoretic conjectures such as $P \neq NP$[3] rule out such mechanisms with useful approximation guarantees in general.[4] Therefore, the community has focused on well-motivated special cases, by assuming some structure on the space of player valuations. Despite intense study over the past decade, however, constant-factor approximation mechanisms for combinatorial auctions have eluded researchers, even for restricted classes of valuations for which (non-incentive-compatible) constant-factor approximation algorithms are known. The most studied such variant, and until recently the most promising candidate for positive results, assumes that player valuations are *submodular*.[5]

There is extensive literature on welfare maximization in combinatorial auctions with submodular valuations. The state of the art in the non-strategic setting, i.e. when incentive-compatibility is dropped as a design requirement, is an $(\frac{e}{e-1})$-approximation algorithm due to Vondrák (2008). A matching impossibility result (Khot et al., 2008) shows this guarantee to be the best possible, assuming $P \neq NP$. Whereas the latter impossibility result applies to a specific class of submodular function with a certain representation, the algorithmic result of Vondrák (2008) holds under very general assumptions on the submodular functions and their representation — namely, that the algorithm has access to a *value oracle* which can be queried for a player's value for any particular bundle. Such an oracle model is a typical abstraction for identifying a class of computational problems of similar complexity, in this case the class of all combinatorial auction problems where players are equipped with valuation functions that are submodular and can be evaluated efficiently, regardless of their representation.[6]

When *both* incentive compatibility and computational efficiency are sought, the outlook for positive results has been more grim. A series of works have provided evidence that computational efficiency and incentive-compatibility are at loggerheads in combinatorial auctions. These works typically make assumptions about the sought mechanisms in three different dimensions: (1) The class of player valuations, most commonly submodular valuations or a close relative[7]; (2) the representation of the valuation functions, either via an oracle model or a chosen explicit representation; and (3) the notion of incentive-compatibility sought, with a focus on how it relates to the use of randomization in the mechanism's allocation and payment rules. For (3), three different restrictions of dominant-strategy incentive compatibility are commonly considered: (a) *truthfulness in expectation* allows a mechanism to randomize its choice of allocation and payments in the interest of computational efficiency, subject to the requirement that truthful reporting maximizes a risk-neutral player's expected payoff regardless of the reports of others[8]; (b) *universal truthfulness* makes no assumptions regarding players' risk attitudes,[9] and requires truth-telling to maximize a player's payoff for *every* draw of the mechanism's internal coins; and (c) *deterministic truthfulness* disallows the use of randomness in the mechanism entirely. Whereas (a), (b) and (c) are progressively more restrictive, the motivation for studying all three is born both of the traditional desire in computer science for understanding the power of randomization as a resource, as well as the fact that theorems regarding mechanisms that restrict the use of randomization have been more technically accessible.

[2] We adopt the standard notion of efficiency used in computer science, namely *polynomial time*. For combinatorial auctions, we say an algorithm runs in polynomial time if it terminates after a number of steps that is polynomial in the number of players and items in the auction.

[3] The class P denotes computational problems that can be solved exactly in time polynomial in the representation size of their input. NP denotes problems for which a correct solution can be *verified* in polynomial time. The class P is known to be contained in NP, and the question of whether this containment is strict is the major open question of computational complexity.

[4] Like much of the related literature, we focus on worst-case multiplicative approximation guarantees in this work. An algorithm or mechanism for combinatorial auctions has an *approximation ratio* of α if it chooses an allocation whose welfare is at least a $1/\alpha$ fraction of the maximum possible for the reported valuations.

[5] Loosely speaking, submodular functions are those set functions that satisfy *diminishing marginal returns*. We define them formally in the preliminaries section.

[6] Other oracle models, such as the *demand oracle* model, or the *communication complexity* model, make stronger assumptions. We refer to the preliminaries section for definitions.

[7] Examples include sub-additive valuations, XOS valuations, and others. For a catalogue of valuations considered in combinatorial auctions, we refer the reader to Blumrosen and Nisan (2007).

[8] We note that truthfulness in expectation is the traditional notion of incentive compatibility considered in much of the economic literature.

[9] Indeed, universal truthfulness goes even further by not requiring that players are expected utility maximizers in the Von Neumann–Morgenstern sense.

Please cite this article in press as: Dughmi, S., Vondrák, J. Limitations of randomized mechanisms for combinatorial auctions. Games Econ. Behav. (2014), http://dx.doi.org/10.1016/j.geb.2014.01.007