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Object allocation via deferred-acceptance: Strategy-proofness and comparative statics ☆

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ABSTRACT

We study the problem of assigning indivisible and heterogeneous objects to agents. Each agent receives at most one object and monetary compensations are not possible. We consider mechanisms satisfying a set of basic properties (*unavailable-type-invariance*, *individual-rationality*, *weak non-wastefulness*, or *truncation-invariance*).

In the house allocation problem, deferred-acceptance (DA)-mechanisms allocate objects based on exogenously fixed priorities over agents. We show that DA-mechanisms are characterized by our basic properties and (i) *strategy-proofness* and *population-monotonicity* or (ii) *strategy-proofness* and *resource-monotonicity*.

Once we allow for multiple identical copies of objects, on the one hand the first characterization breaks down and there are unstable mechanisms satisfying our basic properties and (i) *strategy-proofness* and *population-monotonicity*. On the other hand, our basic properties and (ii) *strategy-proofness* and *resource-monotonicity* characterize (the most general) class of DA-mechanisms based on objects' fixed choice functions that are acceptant, monotonic, substitutable, and consistent.

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1. Introduction

We study the simple model of assigning indivisible and heterogeneous objects (*e.g.*, houses, jobs, offices, school or university admissions, *etc.*) to agents. In many real-life applications economists have recommended practitioners to use (variants of) the deferred-acceptance (DA)-algorithm to assign the objects to agents. Since in our model only one side of the market consists of agents and the other of objects, by DA-algorithm we refer to the agent-proposing DA-algorithm. For the DA-algorithm to be well-defined, objects need to be endowed with priorities over agents in order to determine which agents to reject in case of too many applications. When at most one copy of each object is available, priorities are given by strict orders of agents, and a priority structure is a collection of strict orders, one for each object. Once multiple identical copies of an object are available, priorities are given by choice functions, which choose for any set of agents wanting an object some agents to receive one each (no more than the number of available copies). Choice structures are a collection of choice functions, one for each object. Recent papers (Ehlers et al., 2014; Kamada and Kojima, 2015) constructed choice structures

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taking several important features from applications into account (like controlled school choice constraints or regional caps and target capacities) and then applied the DA-algorithm based on those constructed choice structures.

One key reason for applying the DA-algorithm in real life is that it is *strategy-proof* if the choice structure satisfies certain conditions. In addition, economists have advocated the attractive comparative statics properties of the DA-algorithm (*e.g.*, Crawford, 1991, Hatfield and Milgrom, 2005, and Chambers and Yenmez, 2014): (i) *population-monotonicity*: once some agents leave, all remaining agents weakly benefit from this decrease in competition and (ii) *resource-monotonicity*: once more objects become available, all agents weakly benefit from this increase in resources. Our main results characterize responsive DA-algorithms and choice-based DA-algorithms via a set of basic properties in conjunction with *strategy-proofness* and one of the two comparative statics properties.

We first go back one step and consider the house allocation model[1] where at most one copy of each object is available. We show that in the house allocation model DA-mechanisms with strict priority structures are characterized (in the fixed resources model) by our basic properties, *population-monotonicity*, and *strategy-proofness*. However, this result does not hold in the general model where multiple identical copies of an object may be available and even if we add *truncation-invariance* (the chosen allocation remains unchanged if an agent truncates his preference below his assigned object): there are unstable mechanisms satisfying all our basic properties, *population-monotonicity*, *strategy-proofness*, and *truncation-invariance*. In addition, all these properties minus *strategy-proofness* do not imply stability of the mechanism: the Boston mechanism satisfies all these properties except *strategy-proofness* (but the Boston mechanism is not stable).

Considering the house allocation model with a fixed population and variable resources, our basic properties, *resource-monotonicity*, and *strategy-proofness* do not characterize responsive DA-mechanisms. However, adding *truncation-invariance* to these properties yields another characterization of responsive DA-mechanism via our basic properties, *resource-monotonicity*, *strategy-proofness*, and *truncation-invariance*. In the general model our main result characterizes choice-based DA-algorithms via a set of basic properties in conjunction with *resource-monotonicity*, *strategy-proofness*, and *truncation-invariance*. In addition, we show that any mechanism satisfying all these properties except *strategy-proofness* must be stable, *i.e.*, we must be able to construct a choice structure such that for any problem the mechanism chooses an allocation that is stable with respect to this choice structure.

On the one hand, in the house allocation model *population-monotonicity* is the "stronger" comparative statics requirement because it characterizes responsive DA-mechanisms with our basic properties and *strategy-proofness* whereas *resource-monotonicity* does not (without adding *truncation-invariance*). On the other hand, in the multi-unit allocation model *resource-monotonicity* is the "stronger" comparative statics requirement because it characterizes choice-based DA-mechanisms with our basic properties, *strategy-proofness* and *truncation-invariance* whereas *population-monotonicity* does not.

Therefore, in the general model, choice-based DA-mechanisms are the unique recommendation once we insist in addition to our basic properties on *resource-monotonicity*, *population-monotonicity*, and *strategy-proofness*. Once we drop one of the key properties, choice-based DA-mechanisms remain no longer the *unique* recommendation. This result advocates the DA-algorithm in applications in a stronger fashion than the usual ones whereby a mechanism is advocated because of its properties. The basic properties we use in our characterizations are satisfied by any real-life mechanism we are aware of, *e.g.*, priority mechanisms, the Boston mechanism, linear programming mechanisms, the top trading cycles mechanism, the objects-proposing DA-algorithm, *etc.*

Kojima and Manea (2010) were the first to obtain a characterization of choice-based DA-mechanisms in the general model. They provided two characterizations, (a) one using the two properties of *non-wastefulness* and "individually rational monotonicity" and (b) one using the three properties of *non-wastefulness*, *population-monotonicity*, and "weak Maskin monotonicity". Whereas *non-wastefulness* is a basic requirement, "individual rational monotonicity" and "weak Maskin monotonicity" are new axioms that are arguably more difficult to explain to school boards in policy debates. Ehlers and Klaus (2014) characterize the smaller class of responsive DA-mechanisms whereby each choice function is based on a strict order and chooses from any set of agents the k most preferred elements. In their main result, "two-agent consistent conflict resolution" plays a key role which says that at maximal conflict situations always the same agent should win the conflict (or receive the object). This property is violated by some choice-based DA-mechanisms. Furthermore, in contrast to these characterizations, we show that our main result minus *strategy-proofness* gives us stability, *i.e.*, any such mechanism must be stable with respect to a given choice structure. Note that in all these contributions priorities are derived from a mechanism via a set of properties. Other papers take exogenous priorities as given and impose properties on the mechanism using these exogenous priorities. Balinski and Sönmez (1999) and Morrill (2013) are example of this approach and characterize the deferred-acceptance mechanisms based on "responsive" priorities and "substitutable" priorities.

The paper is organized as follows. In Section 2 we introduce our general object allocation model, properties of mechanisms, and the classes of deferred-acceptance-mechanisms that are either based on acceptant and responsive priority structures or on acceptant, monotonic, substitutable, and consistent choice structures over sets of agents. In Section 3 we state our characterization of responsive-deferred-acceptance mechanisms in the house allocation model using *population-monotonicity* and show how some previous and some new results are implied. Section 4 contains our main result (Theorem 3) for the general object allocation model using *resource-monotonicity*, a characterization of the class of choice-deferred-

[1] The search for "good" mechanisms to solve house allocation problems has been the subject of various contributions (among others): Ehlers (2002), Ehlers and Klaus (2004, 2007, 2011), Kesten (2009), and Pápai (2000).

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