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On bidding with securities: Risk aversion and positive dependence ☆

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ABSTRACT

DeMarzo et al. (2005) consider auctions in which bids are selected from a completely ordered family of securities whose values are tied to the resource being auctioned. The paper defines a notion of relative steepness of families of securities and shows that a steeper family provides greater expected revenue to the seller. Two assumptions are: the buyers are risk neutral; the random variables through which values and signals of the buyers are realized are affiliated. We show that this revenue ranking holds for the second price auction in the case of risk aversion. However, it does not hold if affiliation is relaxed to a less restrictive form of positive dependence, namely first order stochastic dominance (FOSD). We define the relative strong steepness of families of securities and show that it provides a necessary and sufficient condition for comparing two families in the FOSD case. All results extend to the English auction.

1. Introduction

Consider auctioning an asset that is a resource to be developed for profit by the winning buyer. It is common in such auctions to require bids in the form of securities whose values to the seller are tied to the eventual realized value of the asset. As an alternative to simply soliciting cash bids for the asset, for instance, a seller may require buyers to compete in terms of the equity share that the seller retains of the asset's profits. Other common securities used in bidding include debt and call options. DeMarzo et al. (2005) develop a general theory of bidding with securities in the first price and the second price auctions. Bids are selected from a completely ordered family of securities and the paper focuses on the importance of the choice of the family of securities to the seller's expected revenue. The paper defines a partial ordering of families based on the notion of *steepness* (to be made precise in Section 3) and shows that the steeper family of securities provides higher expected revenue to the seller. Two assumptions are made to prove this result: (i) buyers are *risk neutral*; (ii) the random variables through which values and signals of the buyers are realized are *affiliated*. Risk neutrality is a severe restriction for a financial model. Affiliation is an extremely restrictive form of positive dependence.[1]

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[1] de Castro (2010) shows that the set of affiliated probability density functions for two random variables is the complement of an open and dense set in the space of continuous probability density functions under an appropriate topology and has zero measure under an appropriate measure.

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Our objective in this paper is to explore in the case of the second price auction the dependence of the revenue ranking of families of securities upon these two assumptions.[2] We work with a symmetric interdependent values model on the lines of Milgrom and Weber (1982) and risk-averse buyers. We consider two additional forms of positive dependence, namely, a *monotone likelihood ratio* (*MLR*) property, which is weaker than affiliation[3]; and a *first order stochastic dominance* (*FOSD*) property, which is weaker than the MLR property. FOSD captures the idea that the observation by a bidder of a higher signal makes larger values of the other variables more likely. The additional restriction to either MLR or affiliation is attributable to their mathematical value and is typically not motivated in any practical sense. Each of these three positive dependence assumptions has been extensively used in both auction theory and information economics.

Our main results are the following:

(i) A steeper family of securities provides higher expected revenue to the seller even with risk-averse buyers and assuming that the values are positively dependent on signals in the MLR sense. We in this sense extend the result of DeMarzo et al. (2005) to the case of risk aversion and a richer informational environment.
(ii) We show with an example that if the notion of positive dependence among values and signals of buyers is relaxed further from MLR to FOSD, then even for risk-neutral buyers the revenue ranking of families of securities of DeMarzo et al. (2005) no longer holds.
(iii) We strengthen steepness to a property that we call *strong steepness* in order to rank families of securities in the case of FOSD and either risk-neutral or risk-averse buyers. Relative strong steepness is shown to be both necessary and sufficient for comparing two families of securities in this case: one family generates a higher expected revenue for the seller than a second family for all instances of our model satisfying FOSD if and only if it is strongly steeper than the second.
(iv) Finally, we show that the above results extend to the case of the English auction.

It is worth emphasizing that DeMarzo et al. (2005) establish only sufficiency of relative steepness as a condition to rank two families of securities according to the revenue realized by them if affiliation is the notion of positive dependence. By contrast, we show that relative strong steepness is both necessary and sufficient for ranking two families of securities according to the expected revenue realized by them if FOSD is the notion of positive dependence. Furthermore, our proofs are more straightforward than those in DeMarzo et al. (2005) and do not require its strong regularity assumption on the probability density of return conditioned on a buyer's signal. We accomplish this mainly by exploiting the properties of concave functions, which in particular is what allows the consideration of risk-averse buyers in our analysis.

Our paper complements recent work concerning the impact of security choice on the seller's expected profit from auctions. Che and Kim (2010), Kogan and Morgan (2010), and Jun and Wolfstetter (2012) study how the choice of security affects the incentives of the winning bidder in choosing either a level of investment or effort that in turn affects the expected return from the asset. The first case concerns adverse selection while the second concerns moral hazard among bidders. In each case, the ranking of securities based on the seller's net expected profit does not agree with the ranking according to relative steepness in the sense of DeMarzo et al. (2005). None of these three papers, however, explores the effect of risk aversion or the role of the positive dependence assumption in their assessment of security bids.

This paper is organized as follows. Section 2 outlines our model, notation, and definitions. Section 3 extends the revenue ranking of families of securities of DeMarzo et al. (2005) to risk-averse buyers. Section 4 shows that this ranking is not preserved under a more general form of positive dependence, i.e., FOSD. The revenue ranking of families of securities based on strong steepness is then presented. Section 5 provides a brief overview of how the results of Sections 3 and 4 extend to the case of the English auction. We conclude in Section 6.

2. Model, notation, and assumptions

Consider N buyers competing for a resource that a seller wants to sell. Each buyer has a value for the resource that is unknown to him; however, each buyer has some information (*signal*) about the value of the resource. The signal of a buyer is known only to him, but it may be informative to other buyers in the sense that it may improve their respective estimates of the value of the resource.

We model this by assuming that the value of the resource to a buyer n, denoted by x_n, is a realization of a nonnegative random variable X_n, unknown to him. This is the profit to buyer n from developing the resource in the absence of any payments to the seller but after taking into account the variable costs. A buyer n privately observes a signal y_n through a realization of a random variable Y_n that is correlated with $(X_1, X_2, \ldots, X_N)$. A winning buyer needs to invest a fixed

[2] In addition to the second price auction, DeMarzo et al. (2005) also rank families of securities in the case of the first price auction. An additional restriction on the set of securities and the dependence of values and signals beyond affiliation is required in this analysis (i.e., the *log-supermodularity* of each buyer's expected profit, which is Assumption C in the paper). Our interest in this paper is in exploring the effect of relaxing the assumption of affiliation and not restricting it further. We have not been able to carry out the analysis for the first price auction at this level of generality. We do, however, discuss the extension of our results in Section 5 to the commonly used English auction, which is not considered in DeMarzo et al. (2005).

[3] DeMarzo et al. (2005) assume the MLR property for the case of independent private values and affiliation for the case of interdependent values. For independent private values, the MLR property and affiliation are equivalent.