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Bargaining with non-convexities ☆

P. Jean-Jacques Herings *, Arkadi Predtetchinski

Department of Economics, Maastricht University, P.O. Box 616, 6200 MD, Maastricht, The Netherlands

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ABSTRACT

We consider the canonical non-cooperative multilateral bargaining game with a set of feasible payoffs that is closed and comprehensive from below, contains the disagreement point in its interior, and is such that the individually rational payoffs are bounded. We show that a pure stationary subgame perfect equilibrium having the no-delay property exists, even when the space of feasible payoffs is not convex. We also have the converse result that randomization will not be used in this environment in the sense that all stationary subgame perfect equilibria do not involve randomization on the equilibrium path. Nevertheless, mixed strategy profiles can lead to Pareto superior payoffs in the non-convex case.

1. Introduction

Many problems in economics are complicated by the presence of non-convexities. Scarf (1994) mentions the omnipresence of non-divisibilities in production as an important source of non-convexities in economics. Another example of non-convexities in production is the existence of production technologies with increasing returns to scale. Other important cases of non-convexities result from non-convexities in preferences, even in the presence of lotteries when agents are not expected utility maximizers as is for instance the case in prospect theory, see Kahneman and Tversky (1979), or when randomization is not possible, and non-convexities in the consumption set, for instance caused by the presence of indivisible commodities. Although non-convexities are regarded important, most of the economic literature assumes them away for reasons of intractability.

Non-convexities are frequently studied in the n-person cooperative bargaining literature. There is for instance an extensive literature on the extension of the Nash bargaining solution to non-convex environments (Kaneko, 1980; Conley and Wilkie, 1996; Mariotti, 1997; Zhou, 1997; Xu and Yoshihara, 2006).

On the contrary, the literature on strategic bargaining has not paid much attention to non-convexities, and if so, only for the case with two players. Rubinstein (1982) allows for modest forms of non-convexities. Under his hypothesis there is typically a unique subgame perfect equilibrium. Herrero (1989) considers general non-convexities for the two-player case assuming the set of feasible payoffs to be strictly comprehensive and studies the convergence of pure stationary subgame perfect equilibria to the appropriately defined Nash bargaining solution, but does not prove the existence of such equilibria.

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* Corresponding author.
E-mail addresses: P.Herings@maastrichtuniversity.nl (P.J.-J. Herings), A.Predtetchinski@maastrichtuniversity.nl (A. Predtetchinski).

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Conley and Wilkie (1995) also consider a strictly comprehensive set of feasible payoffs and introduce a bargaining protocol that implements their extension of the Nash bargaining solution.

Other notable exceptions are Lang and Rosenthal (2001), In and Serrano (2004), and Shimer (2006). Lang and Rosenthal (2001) study wage bargaining between a union and a single-product firm which needs two types of workers in its production process. The firm bargains with the union over wages for both worker types and is then free to set employment levels to maximize profits. The objective function of the union includes the wage sum and, possibly, employment levels. For a standard Cobb–Douglas production function, it is shown that the resulting set of feasible payoffs is non-convex. In and Serrano (2004) study a bilateral two-issue bargaining procedure with an endogenous agenda. In the procedure, proposals must be made on only one issue at a time, although the proposer can choose which issue to bring to the table. The reduced form of this game, where subgame with a single issue remaining are replaced by the corresponding subgame perfect equilibrium utilities, is one with a non-convex set of feasible payoffs. Shimer (2006) studies a model where a worker is allowed to quit the current job when offered a higher wage by a different employer. Thus when bargaining about the wage, both the worker and the firm take it into account that the employment will terminate as soon as the worker receives a better offer. As a result, the set of expected payoffs feasible for the worker and the firm fails to be convex. In particular, the firm's profit as a function of the wage can be discontinuous at the wage level equal to that of the firm's competitors. All of the above-mentioned papers treat non-convexities in the two-player case only.

Existence of a pure stationary subgame perfect equilibrium in the canonical multilateral bargaining model has only been shown when the set of feasible payoffs is convex. The existence of such an equilibrium has been shown in Banks and Duggan (2000). Merlo and Wilson (1995) consider the n-person cake division problem and obtain the existence of a unique pure stationary subgame perfect equilibrium when the set of feasible payoffs is convex and the proposer selection protocol is deterministic.

We consider the following canonical multilateral bargaining procedure. In each time period, nature randomly selects a player that is allowed to make a proposal. All players respond sequentially to the proposal and either vote in favor or against. As soon as a responder votes against the proposal, the procedure continues in the next period. If all responders vote in favor of the proposal, it is accepted, and the procedure ends. This model is probably the simplest model of multilateral bargaining known in the literature. Merlo and Wilson (1995), Banks and Duggan (2000), Eraslan (2002), Eraslan and Merlo (2002), and Kalandrakis (2006) are some of the many contributions which use a similar or a more general model of multilateral bargaining.

The bargaining game is fully characterized by the set of players, their discount factors, the set of feasible payoffs, and the probability according to which nature selects a particular proposer. The only assumptions we make regarding the set of feasible alternatives are non-substantial technical ones. We normalize the disagreement payoff to be zero and assume the set of feasible payoffs to be closed, comprehensive from below, and the set of non-negative feasible payoffs to be bounded from above. To make the bargaining problem non-trivial, it is moreover assumed that there is an alternative that gives all players a strictly positive payoff.

We show that this entire class of bargaining games has pure stationary subgame perfect equilibria that ensure immediate agreement. This result is surprising as the usual way to deal with non-convexities is to introduce lotteries. For that reason, one might have expected that the equilibria in non-convex bargaining games typically involve mixing. Similarly, one might have expected that non-convexities are a potential source for delay.

We also address the reverse question. Under what conditions are all stationary subgame perfect equilibria of a bargaining game in pure strategies without delay? The answer is that an extremely mild additional assumption assures this: When the set of weakly Pareto optimal alternatives coincides with the set of Pareto optimal ones, all stationary subgame perfect equilibria involve no randomization on the equilibrium path. Equilibria are characterized by the absence of delay.

To derive the first main result, the existence result, we deviate from the usual proof strategy that basically exploits continuity of the best-response correspondences. In our non-convex setting, this correspondence may not be continuous. Instead, we construct an excess utility function that resembles the excess demand function as used in general equilibrium theory.

Let some profile of utilities be given and consider for each player i the (potentially infeasible) proposal player i has to make in order to be consistent with this profile of utilities. Coordinate i of the excess utility function is the degree of feasibility of this proposal. The excess utility function is shown to have a zero point by showing that it is not outward pointing. Next, a zero point is shown to induce a pure stationary subgame perfect equilibrium of the bargaining game.

To prove the second main result, roughly stating that all stationary subgame perfect equilibria are in pure strategies, we proceed in several steps. One of the main steps is to show that in a mixed stationary subgame perfect equilibrium, proposals offering strictly more than the continuation utility to all players are accepted with probability one, whereas proposals offering at least one player strictly less than the continuation utility are accepted with probability zero. The next main step is to argue that for every player there is a unique proposal which maximizes his utility subject to being accepted with probability one and that every mixed stationary subgame perfect equilibrium puts probability one on such a proposal.

Stationary subgame perfect equilibria are efficient in the sense that every proposer selects a weakly Pareto optimal alternative. However, the fact that all equilibria are in pure strategies implies that equilibria may be inefficient in a weaker sense. It is not difficult to construct examples such that the equilibrium utilities are Pareto dominated by the utilities associated with some mixed strategy profiles.