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# Recall and private monitoring ☆

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ABSTRACT

For a general class of games with *private monitoring* we show for any finite state strategy (or automaton strategy) with $D_i$ states for players $i \in \{1, \ldots, N\}$, if there exists a number of periods $t$ such that it is possible on-path to reach any joint state from any joint state in $t$ periods, the strategy is a strict correlated equilibrium only if each player's strategy is a function only of what the player observes in the last $D_i - 1$ periods.


## 1. Introduction

This paper considers a general class of games with *private monitoring* and considers to what extent strategies which depend on outcomes or actions long in the past can be equilibria. The general character of our main results is that strategies which depend on long-ago outcomes or actions are either not equilibria, or "fragile" equilibria which depend on indifference. Examples of such strategies commonly used in games with *public* monitoring include the stick-and-carrot equilibrium in Abreu et al. (1986) and the grim trigger strategy in prisoners-dilemma type games.

Our model considers general games of *private monitoring* where for all actions, all possible private signal profiles occur with positive probability. Further, as in Phelan and Skrzypacz (2012), we limit our analysis to strategies that can be represented by finite automata. For instance, a stick-and-carrot strategy for a quantity setting oligopoly game could have two private states: Punish (where the player chooses a high quantity) and Reward (where the player chooses a low quantity), and where the player transits between Punish and Reward depending on his privately observed price signal. Our equilibrium notion is correlated sequential equilibrium and we ask whether equilibria can be uniformly strict (USCSE), which means that the incentive constraints are satisfied by an amount $\delta > 0$ uniformly for all possible histories. We use correlated sequential equilibrium, as opposed to simply sequential equilibrium, solely for the sake of generality, since sequential equilibria are a special case of the correlated sequential equilibria we consider here.

Our first main result, Proposition 1, states that if the set of possible beliefs a player can have about the state of his opponents while being in one state of his automaton "overlaps" with the corresponding set while being in another state

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of his automaton, then his strategy cannot be a part of a USCSE. The idea behind this result is that the optimality of a player's continuation strategy depends only on his beliefs regarding the continuation strategy of his opponents. If a player's belief sets associated with two distinct states overlap, then there must be two histories, one which puts the player in the first state and another which puts him in the second, which induce (almost) the same beliefs regarding the state of his opponents. Given this, his incentives cannot hold strictly (by an amount uniformly bounded away from zero). In the body of the paper, we show that this result alone is enough to explain why grim-trigger cannot support cooperation (with uniformly strict incentives) in a private monitoring version of the prisoner's dilemma.

In Lemma 1 we show that a strategy represented by a $D_i$-state automaton has either infinite recall or $D_i - 1$ period (or less) period recall (i.e. the action played after any history depends only on what player $i$ observed and has done in the last $D_i - 1$ periods). This is a strengthening of a result in Mailath and Morris (2006).

Finally, Proposition 2 states that if the equilibrium path transition matrix over joint private states of the players is *regular*, (there exists a number of periods $t$ such that it is possible on-path to get from any joint state to any joint state in $t$ periods) then every player $i$'s $D_i$ state automaton must have at most $D_i - 1$ period recall, or the profile of strategies does not form a USCSE, regardless of starting conditions. That theorem applies for example to the stick and carrot type of strategies which describe the best (and worst) equilibria in the public monitoring game of Abreu et al. (1986).

This requirement that the transition matrix of the joint automaton is regular is not necessarily crucial. Many strategy profiles which do not induce regular transition matrices nevertheless share the property that beliefs after long histories converge at least for the relevant priors, as evidenced by the grim trigger example in Section 3.2. The grim trigger automaton does not yield a regular transition matrix since the punish state is absorbing. Yet, if a player starts with an interior belief about his opponent's state, always cooperates and observes a long history of good outcomes, his beliefs will converge to the same interior point no matter what he observed early in the game. As a result, we can find two histories after which player this player has arbitrarily close beliefs about the state of his opponent and yet his strategy calls for different actions. This more general observation is captured by Proposition 1, implying that grim trigger is not a USCSE.

### 1.1. *Relation to previous literature*

This paper contributes to the literature studying strategies instead of payoffs in repeated games with private monitoring. The most closely related paper to ours is Mailath and Morris (2006). In that paper they show that an infinite recall strategy that is a strict perfect public equilibrium of a public monitoring game is no longer a Nash equilibrium if the monitoring is perturbed to be almost-public and rich. There are several differences between our results and theirs. Our results are more limited because we consider uniformly strict sequential equilibria rather than Nash equilibria. On the other hand our results are stronger since, other than full support, we put no conditions on the structure of the monitoring technology (it does not, for instance, need to be almost-public) and we do not require that the profile be an equilibrium (strict or not) of any public monitoring game.

The paper is also suggestive of why existing folk theorems for games with private monitoring either use belief-free strategies that necessarily involve indifference (see for example Ely and Välimäki, 2002, and Ely et al., 2005) or finite recall strategies (see for example Hörner and Olszewski, 2009 or Mailath and Olszewski, 2011). Our results suggest that other equilibria either don't exist, or involve infinite state strategies.[1]

## 2. The model

The underlying model is that same as Phelan and Skrzypacz (2012). The game, $\Gamma^\infty$, is defined by the infinite repetition of a stage game, $\Gamma$, with $N$ players, $i = 1, \ldots, N$, each able to take actions $a_i \in A_i$. With probability $P(y|a)$, a vector of private outcomes $y = (y_1, \ldots, y_N)$ (each $y_i \in Y_i$) is observed conditional on the vector of private actions $a = (a_1, \ldots, a_N)$, where for all $(a, y)$, $P(y|a) > 0$ (*full support*). The sets $A = A_1 \times \ldots \times A_N$ and $Y = Y_1 \times \ldots \times Y_N$ are both assumed to be finite sets. Let $H_i = A_i \times Y_i$.

The current period payoff to player $i$ is denoted $u_i : H_i \to R$. If player $i$ receives payoff stream $\{u_{i,t}\}_{t=0}^\infty$, his lifetime discounted payoff is $(1-\beta)\sum_{t=0}^\infty \beta^t u_{i,t}$ where $\beta \in (0, 1)$. As usual, players care about the expected value of lifetime discounted payoffs.

Let $h_{i,t} = (a_{i,t}, y_{i,t})$ denote player $i$'s private action and outcome at date $t \in \{0, 1, \ldots\}$, and $h_i^t = (h_{i,0}, \ldots, h_{i,t-1})$ denote player $i$'s private history up to, but not including, date $t$. A pure (behavior) *strategy* for player $i$, $\sigma_i = \{\sigma_{i,t}\}_{t=0}^\infty$, is then, for each date $t$, a mapping from player $i$'s private history $h_i^t$, to his action $a_i \in A_i$ in period $t$. Let $\sigma$ denote the joint strategy $\sigma = (\sigma_1, \ldots, \sigma_N)$ and $\sigma_{-i}$ denote the joint strategy of all players other than player $i$, or $\sigma_{-i} = (\sigma_1, \ldots, \sigma_{i-1}, \sigma_{i+1}, \ldots, \sigma_N)$. (Throughout the paper we use notation $-i$ to refer to all players but player $i$.)

### 2.1. *Finite automaton strategies*

As in Phelan and Skrzypacz (2012), we describe strategies in terms of finite automata. (Representing strategies as automata is without loss. The finiteness assumption is restrictive.) Here, since we later restrict discussion to strict equilibria,

[1] See Ely (2002) and Kandori and Obara (2010) for examples of infinite state equilibria.