Games and Economic Behavior 90 (2015) 227–246

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Auctions with online supply

Moshe Babaioff [a], Liad Blumrosen [b,*], Aaron Roth [c]

[a] Microsoft Research, Herzliya, Israel
[b] Department of Economics, The Hebrew University, Jerusalem 91905, Israel
[c] Computer and Information Sciences, University of Pennsylvania, Philadelphia, PA 19104, USA

ARTICLE INFO

Article history:
Received 12 June 2011
Available online 18 February 2015

JEL classification:
D44
D82
D83

Keywords:
Dynamic auctions
Unknown supply
Online auctions
Approximation
Dynamic mechanism design
Stochastic supply

ABSTRACT

Online advertising auctions present settings in which there is uncertainty about the number of items for sale. We study mechanisms for selling identical items when the total supply is unknown but is drawn from a known distribution. Items arrive dynamically, and the seller must make immediate allocation and payment decisions with the goal of maximizing social welfare. We devise a simple incentive-compatible mechanism that guarantees some constant fraction of the first-best solution. A surprising feature of our mechanism is that it artificially limits supply, and we show that limiting the supply is essential for obtaining high social welfare. Although common when maximizing revenue, commitment to limit the supply is less intuitive when maximizing social welfare. The performance guarantee of our mechanism is in expectation over the supply distribution; We show that obtaining similar performance guarantee for every realization of supply is impossible.

1. Introduction

Consider the problem that an online advertising marketplace faces when selling banner advertisements. The items being sold are “page impressions” of a particular banner-ad slot, and they arrive whenever a new user navigates to the website in question. The items being sold are identical,[1] and in a first cut approximation, the set of bidders in this auction is fixed.[2] On the other hand, the *supply* of items is dynamic: new page impressions are constantly arriving, and although the market maker might have distributional information about the number of impressions he expects to receive in a day, he does not know this quantity with certainty ahead of time. Moreover, these items are perishable: page views cannot be stored and saved for later allocation. Therefore, items must be allocated immediately when they arrive, without waiting for information about future supply. Finally, since the market is essentially ongoing, and it is not possible to wait indefinitely for payments, bidder payments must also be computed before the final supply is fully realized.

* Corresponding author.
E-mail addresses: moshe@microsoft.com (M. Babaioff), blumrosen@huji.ac.il (L. Blumrosen), aaroth@cis.upenn.edu (A. Roth).

[1] Each ad impression is identical from the advertiser's perspective in the absence of knowledge about who is viewing the website. It is true that advertisers sometimes have coarse information about users by buying cookie-data from providers such as Acxiom, but in this case we can simply restrict our attention to ad impressions viewed by a particular fixed demographic.

[2] Different advertisers can of course enter and leave the market for a particular banner ad slot, but this happens at a human time scale, whereas ad impressions arrive at a time scale of milliseconds.

http://dx.doi.org/10.1016/j.geb.2015.01.004
0899-8256/

This paper considers a natural abstraction of dynamic mechanism design with the goal of capturing the key properties of the advertisement market described above, along with other kinds of markets with similar considerations.[3] In contrast to previous work on dynamic auctions which have focused on dynamic bidder arrival (see e.g. the recent surveys on dynamic auctions by Parkes, 2007 and Bergemann and Said, 2010), we have a fixed set of n bidders known to the mechanism, who each have a private value for obtaining a single item. There is an unknown supply of ℓ identical items that arrive one at a time (i.e. ℓ is not known to the mechanism). Finally, we require that the mechanism be *prompt* – that it make allocation and payment decisions immediately when each item arrives, without waiting to see if another item is coming. In this model, we investigate when it is possible for mechanisms to obtain a significant fraction of the social welfare obtainable when the supply is known in advance (the "optimal social welfare").

We study this model with an eye towards one of the "holy grails" of the mechanism-design literature – the design of "detail-free" mechanisms, along the lines of the *Wilson Doctrine* (Wilson, 1987). The ideal goal is to design robust mechanisms which need not be aware of specific characteristics of the environment, and in particular, should not rely on common-prior assumptions. Specifically, we wish to avoid assuming the existence of a known prior on bidder valuations, since it is not clear that an advertising platform can get good information on the valuations of bidders, who may be heterogeneous and anonymous. Specifically, we ask whether any mechanism exists which can guarantee a reasonable fraction of the social welfare when the mechanism has no distributional information about bidder valuations. To formalize the notion of "a reasonable fraction of the social welfare", we borrow a robust asymptotic dichotomy that has proven extremely useful in the computer science literature. We think of social welfare as being approximable in a given setting if there exists some ex-post truthful mechanism which guarantees to always achieve some constant fraction of the social welfare. In contrast, we think of social welfare as being in-approximable if every ex-post truthful mechanism achieves only a diminishing fraction of the optimal social welfare in the worst case. We show that even in the absence of any distributional information about bidder valuations, it is possible to approximate the optimal social welfare when supply is uncertain. Our algorithms do rely on distributional information about the supply. This is more reasonable in our setting, because the auctioneer might have extensive history selling page views of a given website. Moreover, we show that distributional information about the supply is necessary: no mechanism can guarantee a non-diminishing fraction of the optimal social welfare in the worst-case over the realization of the supply.

Our positive results highlight a novel phenomenon: in the presence of uncertainty about the supply, it is necessary to artificially *restrict* supply in order to approximate social welfare. Artificially restricting supply is a common technique when the goal is to maximize revenue, but in our setting, it is a surprising necessity, given that our goal is welfare maximization.

1.1. Our results

We consider the stochastic-supply setting in which supply is drawn from a distribution D known to the mechanism, and welfare guarantees are required to hold in expectation over D. Throughout this paper, we study mechanisms which are ex-post dominant strategy truthful (i.e. truthful for every realization of the unknown supply).[4] We make the assumption (standard in mechanism design in the context of bidder valuations, but also natural here) that D has a non-decreasing hazard rate.[5] We again stress that we make *no* assumptions about bidder valuations at all. We obtain a positive result:

Theorem. *There exists a truthful mechanism that achieves a constant approximation to social welfare when supply is drawn from a known distribution with non-decreasing hazard rate.*

This mechanism is simple, deterministic, computationally efficient, and easy to implement, but it's analysis is surprisingly subtle. As noted, the incentive properties of our mechanisms do not rely on any distributional information and truthful bidding is a dominant strategy for every set of bids, for *every* realized supply and not only in expectation. Surprisingly, our mechanism relies on restricting supply, which is unusual in a welfare-maximization setting.

We then characterize the set of truthful mechanisms that are constrained to collect payments as items are allocated, and prove a surprising lemma (see Lemma 3.4, Section 3): in the stochastic setting, we can (almost) without loss of generality consider mechanisms that determine an upper bound on the number of items to be sold without considering the bids. More specifically, we observe that in a setting with uncertainty over supply, a mechanism that does not limit the supply can obtain an arbitrarily bad approximation to social welfare. We might a priori think that in order to achieve a good approximation, a mechanism may have to limit the supply in a way that depends on some arbitrarily complicated function of the bids. However, Lemma 3.4 shows that this is not the case: for any truthful mechanism, there is another truthful mecha-

[3] Uncertainty on the supply appears in various environments. More examples include markets for computing resources and also traditional markets, like agricultural markets, where produce and fish continue to arrive after markets has been opened.

[4] The weaker solution concept of truthfulness in expectation over the realization of the supply would require that we assume that the bidders are risk neutral. We note that if we only required truthfulness for risk-neutral bidders, then in the stochastic setting we could obtain optimal social welfare by merely charging every agent their "expected" VCG price – but in addition to risk neutrality, this would require an assumption that all bidders agree with the mechanism on the prior distribution over supply. We view these as needlessly strong assumptions, especially in light of our positive results.

[5] A cumulative distribution F with density f has *non-decreasing hazard rate* (sometimes called *monotone hazard rate*) if $\frac{f(x)}{1-F(x)}$ is non-decreasing with x.