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Note

Sequential cheap talks

Keiichi Kawai [1]

University of New South Wales, UNSW Business School, Sydney, NSW 2052, Australia

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ABSTRACT

In this note, we analyze a multidimensional cheap talk game where two senders sequentially submit messages. We provide a necessary and sufficient condition for the existence of a fully-revealing equilibrium.

1. Introduction

In this note, we analyze a multidimensional cheap talk game where two senders sequentially submit messages. We show that a fully-revealing equilibrium exists if and only if the senders' biases are opposing. That is, the product of the vectors that represent senders' biases are negative.

As shown in Krishna and Morgan (2001), when the state space is one-dimensional Euclidean space, a fully-revealing equilibrium exists if and only if two senders' biases are opposing. We show that the analysis of Krishna and Morgan (2001) for the one-dimensional Euclidean space can easily be extended to a general n-dimensional Euclidean state space.

It is well known that multidimensional cheap talk games have positive results on information transmission when the senders simultaneously send messages.[2] When messages are submitted sequentially, one might think it is impossible to achieve the fully-revealing equilibrium, even when the biases are opposing, because there exists an action that both senders strictly prefer over the receiver's ideal action.[3]

We nonetheless show that a fully-revealing equilibrium exists if and only if the senders' biases are opposing. The result follows from an observation that the conflict of interests between the decision maker and the second sender is essentially one-dimensional. More precisely, we can always transform the coordinate system so that the decision maker's actions that are ideal for the decision maker and the second sender differ in only one coordinate.

This observation implies that the existence of a fully-revealing equilibrium boils down to whether or not the decision maker can successfully solicit information regarding the coordinate on which the decision maker and the second sender have conflict of interests. This in turn enables us to fully utilize the concept of self-serving messages characterized in Krishna and Morgan (2001), i.e., the set of second sender's messages that the decision maker disregards. As a result, the set

E-mail address: k.kawai@unsw.edu.au.

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[2] See Battaglini (2002).

[3] Note that such an action does not exist when the state space is one-dimensional.

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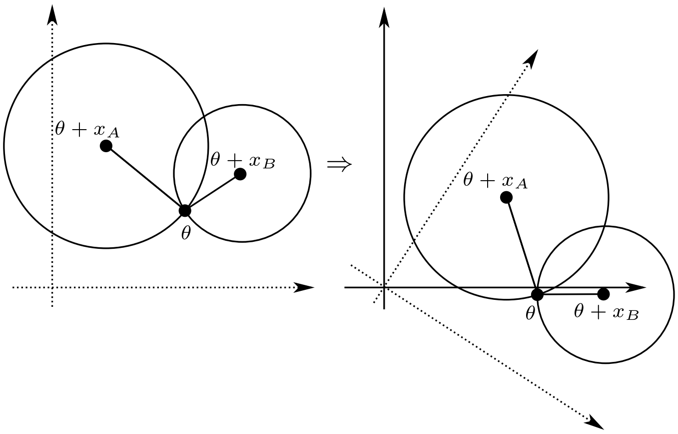

Fig. 1. Normalization of x_B.

of second sender's messages that the decision maker disregards in this note has a direct analogy to the one-dimensional Euclidean space model.

This is a clear contrast to the constructive proof by Miura (2014) that has independently shown the same result as in this note. In Miura (2014), the construction of the set of second sender's messages that the decision maker disregards heavily relies on the nature of quadratic-loss payoff functions. Consequently, the analysis in Miura (2014) has two limitations. First, it does not provide a direct analogy to the one-dimensional Euclidean space model. Moreover, it is only applicable to quadratic-loss payoff functions. In contrast, even though we only present the result for the quadratic-loss payoff functions, the analysis presented in this note can easily be generalized to more general payoff functions.

2. Model

We consider the following n-dimensional cheap talk game with sequential communication. There are three players: two senders, A and B, and a decision maker (DM, henceforth). By $u_i^\theta(a)$, we denote player i's payoff when the true state is θ and action taken by the DM is a. We assume that both the state space and action space of the DM are $\mathbb{R}^n$, and payoff functions are quadratic-loss, that is, $u_i^\theta(a) = -\sum_{j=1}^n (a - (\theta + x_i))^2$, $i = DM, A, B$, and $x_{DM} = 0$.[4]

The game proceeds as follows:

1. Senders learn the true state θ;
2. Sender A sends a message $m_A : \theta \mapsto m_A \in \mathbb{R}^n$;
3. Sender B sends a message $m_B : \theta \times m_A \mapsto m_B \in \mathbb{R}^n$;
4. DM takes an action $a : m_A \times m_B \mapsto a \in \mathbb{R}^n$.

The equilibrium concept we use is a perfect Bayesian equilibrium.

3. Fully-revealing equilibrium

For the rest of the paper, we normalize $x_B = (x_{B1}, 0, 0, \cdots, 0)$, $x_{B1} \geq 0$. That is, we use an orthogonal basis so that the conflict of interests between the DM and Sender B is about the first-dimension alone. This normalization is without loss of generality, but simplifies notations.[5]

With such a normalization, the existence of a fully-revealing equilibrium boils down to the question of whether the DM can elicit the truthful report with respect to the first-dimension. As it turns out, a fully-revealing equilibrium exists if and only if two senders' biases are **opposing**, i.e., $x_A \cdot x_B \leq 0$.

Theorem 1. *There exists a fully-revealing equilibrium if and only if* $x_A \cdot x_B \leq 0$.

To understand the intuition behind the "if" part of the result, it is useful to review the case where $n = 1$. As shown in Krishna and Morgan (2001), a fully-revealing equilibrium exists if and only if $x_A < 0 \leq x_B$, i.e., $x_A \cdot x_B \leq 0$. A strategy profile that supports the fully-revealing equilibrium is as follows:

[4] The result in this note can be easily generalized for more general payoff functions. But for expositional simplicity, we limit our attention to the quadratic-loss payoff functions.

[5] This can be done by simply "rotating" the coordinates. See Fig. 1 for $n = 2$ case.