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Time and Nash implementation

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ABSTRACT

In this paper, we study the full implementation problem using mechanisms that allow a delay. The delay on the equilibrium path may be zero, an infinitesimally small number or a fixed positive number. In all these three cases, implementable rules are fully characterized by a monotonicity condition. We provide examples to show that some delayed implementable social choice rules are not implementable in Nash-equilibrium refinements without a delay. As an application of our approach, we characterize delayed implementable rules in environments where only the discounting changes between states.

1. Introduction

In this paper, we develop a general theory of mechanisms that use delay in the context of full implementation in Nash equilibrium. Many real-life mechanisms explicitly allow for a delay; some of the most prominent examples are the rule permitting filibusters in the United States Senate and suspensive veto power in the British parliament.[1]

We assume that delay is undesirable and is limited by a maximum allowable delay T. We study three distinct restrictions for a delay Δ to occur on the equilibrium path: (i) $\Delta = 0$; (ii) Δ is arbitrarily small, and (iii) Δ is a fixed number between zero and T.

Under restriction (i), $\Delta = 0$, delay can occur off the equilibrium path; this is the most restrictive case for delayed implementation. For this case, we construct a novel canonical mechanism that functions without relying on either no-veto-power conditions, or restrictions on the environment beyond those implied by an added time dimension.

We call implementation under restriction (ii) *imminent implementation*. Restriction (ii) allows us to deal with indifference in agents' preferences. In particular, we show in Examples 1 and 2 that some SCRs are imminently implementable, but not implementable in a Nash equilibrium or its refinements.

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[1] Filibuster refers to a procedure that allows senators to make speeches and amendments indefinitely, thus delaying a vote. Suspensive veto power allows the House of Lords to delay the enactment of a law by one year. Mayhew (2003) argues that filibuster can be explained as a tool to test resoluteness of the parties. Wawro and Schickler (2006) elaborate on this argument.

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Under restriction (iii), delayed implementation with $\Delta > 0$ allows us to exploit preference reversals that happen only "at a later time," that is, around a social outcome $(a, t + \Delta)$, but not around (a, t). This is the most permissive case. As an illustration, we consider an environment where only time preferences change and show, in Section 5, that if there are "sufficiently many" outcomes, even a small delay would allow any SCR to be Δ-delayed implementable.

Under the restriction that there is no delay on the equilibrium path, $\Delta = 0$, our problem is similar to the problem studied by Bochet (2007), Benoit and Ok (2008), and Sanver (2006).

Bochet (2007) and Benoit and Ok (2008, Section 3) augment their environments by lotteries instead of time. Both papers impose additional conditions on the environment (strict preferences and top coincidence, respectively), that are not required in this paper. The canonical mechanisms constructed in these two papers also differ in an essential way from the mechanism constructed here, which exploits the undesirability of delay.

Benoit and Ok (2008, Section 4) and Sanver (2006) augment an environment by allowing the designer to give a payment (an award) to an agent off the equilibrium path. There are two important differences between an award and a delay. First, an award affects a single agent, while delay affects all agents. Separability has been shown to be important in other implementation settings (Kunimoto and Serrano, 2011). Second, no-veto-power is vacuously satisfied in an environment with awards. As this is not the case under our restriction (i), our mechanism that implements an SCR with zero delay needs to account for this condition.

The necessary and sufficient condition derived by Sanver (2006) for implementation by awards is weaker than our condition for zero-delayed implementation (see Example 1 in this paper). When an infinitesimally small delay is allowed on the equilibrium path (our restriction (ii)), our condition becomes very similar to Sanver's.[2]

In allowing to approximate an SCR, imminent implementation is the most similar to virtual implementation. Abreu and Sen (1991) and Matsushima (1988) show that any SCR can be virtually implemented; the results of this paper are not nearly as permissive. A mechanism that virtually implements an SCR delivers the social outcome with an arbitrarily high probability, but it relies critically on an occasional delivery of an incorrect outcome. This may be a serious practical drawback; arguments against schemes that deliver socially suboptimal outcomes ex-post are abundant in legal scholarly papers (Fried, 2003). To give a particularly striking example, a virtual mechanism for King Solomon's problem of allocating a child to a true mother assigns positive probability to killing the baby *even if* King Solomon successfully determines who the true mother is (Serrano, 2004). A mechanism that imminently implements the King Solomon problem relies on King Solomon having custody over the baby over a short period of time (Artemov, 2006).[3] We think that the latter inefficiency is more tolerable for society than the former.

In the next section, we define Δ-delayed implementation. In Section 3 we construct a canonical mechanism and provide the characterization of delayed implementable SCRs. We collect examples in Section 4. Section 5 contains an application of our characterization to the environment in which only time preferences change. We conclude in Section 6.

2. Preliminaries

There is a finite set $N = \{1, \ldots, n\}$ of agents and a known set A of *physical* outcomes. Let the set of outcomes $\mathcal{A} = A \times [0, T]$ be the set of physical outcomes augmented by time. The value $T > 0$ is an upper bound on a delay. If an arbitrary delay is allowed, then $\mathcal{A} = A \times \Re_+$. We interpret an element $(a, t) \in \mathcal{A}$ as a physical outcome a delivered at time t.

Let Θ be the finite set of possible states. Associated with each $\theta \in \Theta$, there is a preference profile $\succeq^\theta = (\succeq_1^\theta, \ldots, \succeq_n^\theta)$, where $\succeq_i^\theta$ represents agent i's preference ordering over $\mathcal{A}$. The symbols $\succ_i^\theta$ and $\sim_i^\theta$ represent a strict preference relation and indifference, respectively. We assume throughout the paper that preferences are complete and transitive. Let $A \subseteq \Theta$ and $B \subseteq N$; then $(a, t) \succ_B^A (b, t')$ means that for every state $\theta \in A$ and every agent $i \in B$, $(a, t) \succ_i^\theta (b, t')$. The notation $\succeq_B^A$ and $\sim_B^A$ is defined analogously.

We make two assumptions on time preferences: (1) the strict undesirability of delay and (2) continuity. Formally, we assume that for any $\theta \in \Theta$ and any $i \in N$, the following conditions hold: (1) for any $(a, t) \in \mathcal{A}$, and any $t' > t$, $(a, t) \succ_i^\theta (a, t')$; and (2) the upper contour set $\{(b, t') \in \mathcal{A} | \forall (a, t) \in \mathcal{A}, (b, t') \succeq_i^\theta (a, t)\}$ and the lower contour set $\{(b, t') \in \mathcal{A} | \forall (a, t) \in \mathcal{A}, (a, t) \succeq_i^\theta (b, t')\}$ are closed.

A *social choice rule* (SCR) $F : \Theta \to 2^{\mathcal{A}} \setminus \emptyset$ is a mapping from the set of states Θ to a subset of outcomes in $\mathcal{A}$.

Definition 1. An SCR F^Δ is Δ-*delayed* with respect to an SCR F if, for any θ, the following two conditions hold:

1. For all $(a, t) \in F(\theta)$, there exists $t' \in [t, t + \Delta] \cap [0, T]$ such that $(a, t') \in F^\Delta(\theta)$ and
2. For all $(a, t') \in F^\Delta(\theta)$, there exists $t \in [t' - \Delta, t'] \cap [0, T]$ such that $(a, t) \in F(\theta)$.

That is, a Δ-delayed SCR delivers the same physical outcome with a delay of no more than Δ relative to the original delay. We assume throughout the paper that $\Delta \geq 0$.

[2] To be more precise, if this paper were to address Sanver's question and require implementation for any possible specification of preferences over time, the conditions would have been identical.

[3] Sanver (2004) uses the King Solomon problem to point out that adding outcomes to the environment may aid monotonicity.