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Note

Interim partially correlated rationalizability

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ABSTRACT

We formalize a solution concept called interim partially correlated rationalizability (IPCR), which was implicitly discussed in both Ely and Peski (2006) and Dekel et al. (2007). IPCR allows for interim correlations, i.e., correlations that depend on opponents' types but not on the state of nature. As a direct extension of Ely and Peski's main result, we show that hierarchies of beliefs over conditional beliefs are necessary and sufficient for the identification of IPCR. We use new proof techniques that better illustrate the connection between higher order beliefs and interim rationalizability.

1. Introduction

We follow the literature that studies the connection between solution concepts and information modeling in games with incomplete information. Conventionally, modelers choose hierarchies of beliefs about payoffs as the primitives of the game and use Harsanyi type spaces (Harsanyi, 1967–1968) to model them. Solution concepts are then defined on type spaces. However, as is commonly known in the literature, under a fixed payoff structure of a game, type spaces that represent the same set of hierarchies of beliefs may give different Bayes Nash equilibrium predictions.[1]

Two approaches are taken to restore the connection between solution concepts and information. Dekel et al. (2007) (hereafter, DFM) define interim correlated rationalizability (hereafter, ICR) and show that the conventional Mertens–Zamir hierarchy of beliefs of a type is sufficient for identifying the set of ICR actions of that type.[2] In their definition, a player may conjecture that her opponents' types, states of nature and opponents' actions could be arbitrarily correlated, as long as the correlation is consistent with her belief in the type space. In a parallel work, Ely and Peski (2006) (hereafter, EP) study interim independent rationalizability (hereafter, IIR) for two-player games, in which each player assumes that the opponent's action correlates with state of nature only through the opponent's type. They introduce Δ-hierarchy of beliefs which contains weakly richer information than conventional hierarchy of beliefs of any type and show that it is necessary and sufficient for IIR.

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[1] For example, any correlated equilibrium in a complete information game can be viewed as a type space. On all such type profiles, players have common knowledge of the game. However, there may be action profiles played in Bayes Nash equilibrium but not Nash equilibrium of the complete information game.

[2] The necessity condition is given in Dekel et al. (2006).

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When there are more than two players, a player could conjecture that there are correlations among other players' strategies.[3] Unlike in games with complete information where the correlation takes a natural form, for games with incomplete information, there are multiple possibilities for correlations, depending on which type of information is given as the primitive. In particular, there are multiple ways of extending EP's formulation to games with more than two players, as is also discussed in Ely and Peski (2006), Section 7.2.

Following EP, we define interim partially correlated rationalizability (hereafter, IPCR) for n-player games by considering conjectures that involve only interim stage correlations. We assume that players view opponents' actions as type-contingent variables, and that a player's conjecture over opponents' actions and state of nature is induced by her belief in the type space together with a type-correlated strategy of the opponents'. A type-correlated strategy of the opponents' maps each profile of their types to a probability measure on their action profiles. If we take the agent-normal-form view of a type space, i.e., if we view each type of a player as an agent of that player, then the correlation in IPCR is similar to that permitted in the definition of correlated rationalizability for complete information games played by agents. We may also view the correlation we permit as interim correlation, while view that permitted by DFM as ex post correlation.

Our main result is a direct extension of EP's main theorem, in which we show that two types have the same interim partially correlated rationalizable behavior if and only if they have the same Δ-hierarchy of beliefs. In the n-player environment, Δ-hierarchy of beliefs is defined as the hierarchy of beliefs over conditional beliefs, where each conditional belief of a player is her belief over states of nature given a profile of her opponents' types. This result justifies the definition of IPCR, and also provides another connection between solution concepts and information.

Why do we need richer information to identify IPCR? Similar to the case of IIR, it is because conventional hierarchy of beliefs fails to capture certain correlations that potentially affect IPCR. We illustrate this with a simple but non-degenerate three-player game with incomplete information. This example also suggests the necessity of beliefs over conditional beliefs.[4]

Example 1. Let $\Theta = \{+1, -1\}$ be the set of states of nature. Consider two type spaces that both model common knowledge of equal probability on $\theta = +1$ and $\theta = -1$. In type space T, $T_1 = T_2 = T_3 = \{*\}$, and the common prior is $\mu[\theta = +1] = \mu[\theta = -1] = \frac{1}{2}$. In type space $\hat{T}$, $\hat{T}_1 = \{*\}$, $\hat{T}_2 = \hat{T}_3 = \{+1, -1\}$, and the common prior $\hat{\mu}$ on $\hat{T}_2 \times \hat{T}_3 \times \Theta$ is given by

$\theta = +1$:

$T_2 \backslash T_3$	$+1$	-1
$+1$	$\frac{1}{4}$	0
-1	0	$\frac{1}{4}$

$\theta = -1$:

$T_2 \backslash T_3$	$+1$	-1
$+1$	0	$\frac{1}{4}$
-1	$\frac{1}{4}$	0

Suppose player 2 and player 3 are trying to coordinate their actions on the right state of nature and their payoffs (given in the tables below) do not depend on player 1's action.

$\theta = +1$:

	a_3	b_3
a_2	1, 1	0, 0
b_2	0, 0	1, 1

$\theta = -1$:

	a_3	b_3
a_2	0, 0	1, 1
b_2	1, 1	0, 0

Player 1, instead, chooses between *Bet* and *Not Bet*. By choosing *Bet*, she receives a payoff of 1 when the other two successfully coordinate on the right state θ; otherwise she receives 0. By choosing *Not Bet*, she always receives $\alpha \in (\frac{1}{2}, 1)$.

We use IPCR as the solution concept. When the type space is T, for example, in Player 1's conjecture, players 2 and 3 can play any type-correlated strategy $\sigma_{2,3} : T_2 \times T_3 \to \Delta(\{a_2, b_2\} \times \{a_3, b_3\})$,[5] but their strategies cannot directly depend on θ, the true state of nature. Note that at type space T, for any conjecture of player 1, her expected payoff from choosing *Bet* is $\frac{1}{2}$, less than α, her payoff from choosing *Not Bet*. Thus, the action *Bet* is not rationalizable for her at type space T. When the type space is $\hat{T}$, however, player 1 may justifiably conjecture that both opponents choose a at type $+1$ and choose b at type -1, thus successfully coordinate at both states of nature. Under this conjecture, player 1 receives $1 > \alpha$ by choosing *Bet*. That is, at type space $\hat{T}$, *Bet* becomes rationalizable for player 1.

Hence we see that at both type spaces, player 1 has the same conventional hierarchy of beliefs, but has different sets of IPCR actions. Essentially, players 2 and 3 can coordinate better at type space $\hat{T}$ because their types in $\hat{T}$ involve more correlation with θ.[6] We show explicitly in Example 2 that such correlation is captured by player 1's hierarchy of beliefs over conditional beliefs.

[3] Such correlations are familiar to us and are well-studied for games with complete information under correlated equilibrium (Aumann, 1974, and Aumann, 1987) and correlated rationalizability (Bernheim, 1984; Pearce, 1984, and Brandenburger and Dekel, 1987).

[4] This example incorporates elements from the motivating examples of both Ely and Peski (2006) and Liu (2015).

[5] For any complete separable metric space X, ΔX is the space of Borel probability measures on X.

[6] In fact, type space $\hat{T}$ can be generated through type space T using a simple state-dependent correlating device, which does not change players' hierarchies of beliefs (Liu, 2015). However, if such a device is not a partially correlating device, then it changes players' hierarchies of beliefs over conditional beliefs (Tang, 2015). Please see Section 4 for more discussion.