Games and Economic Behavior 86 (2014) 26–39

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Impossibility results for parametrized notions of efficiency and strategy-proofness in exchange economies ☆

Wonki Jo Cho

School of Social Sciences, University of Manchester, Oxford Road, Manchester, M13 9PL, United Kingdom

ARTICLE INFO

Article history:
Received 15 January 2013
Available online 18 March 2014

JEL classification:
D71

Keywords:
Exchange economy
Individual rationality
δ-Efficiency
δ-Strategy-proofness

ABSTRACT

We study a standard model of exchange economies with individual endowments. It is well known that no rule is *individually rational*, *efficient*, and *strategy-proof*. In order to quantify the extent of this impossibility, we parametrize axioms on allocation rules. Given an axiom A, a parametrization of A is a continuum of axioms $\{\delta\text{-}A\}_{\delta\in[0,1]}$ such that (i) δ-A is equivalent to A if and only if $\delta = 1$; (ii) δ-A is vacuous if and only if $\delta = 0$; and (iii) for each pair $\delta, \delta' \in [0, 1]$ with $\delta < \delta'$, δ'-A implies δ-A. Thus, as δ decreases from 1 to 0, δ-A weakens monotonically, eventually to a vacuous requirement. We consider two parametrizations $\{\delta\text{-}\mathit{efficiency}\}_{\delta\in[0,1]}$ and $\{\delta\text{-}\mathit{strategy\text{-}proofness}\}_{\delta\in[0,1]}$, and investigate their compatibility with *individual rationality* for the class of two-agent economies defined on the linear preference domain. We show that (i) for each $\delta \in (0, 1]$, no rule is *individually rational*, *δ-efficient*, and *strategy-proof*; and (ii) for each $\delta \in (0, 1]$, no rule is *individually rational*, *efficient*, and *δ-strategy-proof*.

1. Introduction

We study standard exchange economies with individual endowments. There is a set of perfectly divisible commodities. Each agent has a preference relation defined over non-negative amounts of those commodities. He also owns some amounts of the commodities, which we call his (individual) endowment. An economy is a profile of preference relations and endowments, and an allocation for the economy is a profile of (consumption) bundles whose sum is equal to the sum of the endowments. An (allocation) rule assigns to each economy an allocation for it.

We search for rules satisfying some desirable properties, or axioms, and the following three axioms have long dominated the literature on this quest: (i) *individual rationality*, the requirement that for each economy, a rule assign to each agent a bundle that he finds at least as desirable as his endowment; (ii) *efficiency*, the requirement that for each economy, a rule select an allocation such that no other allocation Pareto dominates it; and (iii) *strategy-proofness*, the requirement that a rule select allocations in such a way that no agent ever benefits from lying about his preference relation.

The three requirements, however, are incompatible. Hurwicz (1972) shows that for the class of two-agent and two-commodity economies, no rule meets all of them.[1] Subsequent studies strengthen this theorem mainly in two directions.[2]

☆ This paper is based on a chapter in my Ph.D. dissertation at the University of Rochester. I am deeply grateful to William Thomson for his guidance and encouragement. I also thank Hye Jin Ahn, an associate editor, anonymous referees for comments that greatly improved the earlier version of this paper.

E-mail address: jo.cho@manchester.ac.uk.

[1] Hurwicz's (1972) theorem covers only two-agent and two-commodity economies, and Serizawa (2002) generalizes it to economies with an arbitrary number of agents and an arbitrary number of commodities.

[2] Most papers cited here pursue the two directions simultaneously.

http://dx.doi.org/10.1016/j.geb.2014.03.003
0899-8256/

The first strengthening involves establishing similar results on smaller preference domains. Hurwicz (1972) works with classical preference relations (i.e., those that are continuous, monotone, and strictly convex). As it turns out, this is quite a rich domain, and his impossibility theorem still holds when agents' preference relations are restricted to a much smaller subset. Parallel results are available on the domain of linear preference relations (Schummer, 1997) and the domain of CES preference relations (Ju, 2003).

The second way of strengthening is to show that each *efficient* and *strategy-proof* rule violates some fairness axiom, and then obtain Hurwicz's (1972) theorem as a corollary. A number of authors consider the following as fairness criteria: (i) *non-dictatorship*, the requirement that there be no agent who receives everything in each economy (Dasgupta et al., 1979; Ju, 2003; Schummer, 1997); (ii) *non–inverse-dictatorship*, the requirement that there be no agent who receives nothing in each economy (Zhou, 1991); and (iii) *minimum consumption guarantee*, the requirement that each agent receive a bundle bounded away from the origin (Serizawa and Weymark, 2003).

Among this range of stronger impossibility results, absent is a theorem that shows the incompatibility of *individual rationality* with weaker versions of *efficiency* and *strategy-proofness*.[3] We attribute the absence to the strong normative appeal of the latter two properties. Viewed separately, *efficiency* is so mild a requirement that weakening it appears hardly necessary (no economist would object to making one agent better off without hurting any other agent). On the other hand, *strategy-proofness*, though demanding, is an axiom that we cannot dispense with in the context where agents' private information, e.g., preference relations, should be elicited. On these grounds, the two axioms are widely accepted, to the extent that most axiomatic analyses take them as "basic" requirements and study the consequences of imposing some other axioms additionally.

Our motivation to weaken *efficiency* and *strategy-proofness* goes beyond theoretical interest and is based on the following scenario we model. A group of agents, each with an endowment, gather to find an allocation that is beneficial to all. The endowments are privately owned, and we operationalize the notion of private ownership by giving each agent the right to consume his endowment. Then as the agents negotiate on who gets what, their endowments serve as an important benchmark: whenever the collective decision assigns an agent a bundle less desirable than his endowment, he can simply walk out. In situations like this, *individual rationality* is an axiom that should be met first, and Hurwicz (1972) shows that in its presence, we cannot have both *efficiency* and *strategy-proofness*. Then how much *efficiency* should we sacrifice for *individual rationality* and *strategy-proofness*? Or how much *strategy-proofness* should we abandon for *individual rationality* and *efficiency*? These are the questions we address.

Our contribution consists in (i) providing weakenings of *efficiency* and *strategy-proofness*—in fact, parametrizations thereof; and (ii) showing that when combined with *individual rationality*, either of the two axioms forces a rule to satisfy only the vacuous version of the remaining axiom. Before introducing our parametrizations, let us first explain an underlying principle. Let A be an axiom. Let $[0,1]$ be the parameter space and δ the parameter. A parametrization of A is a continuum of axioms $\{\delta\text{-}A\}_{\delta\in[0,1]}$ such that (i) δ-A is equivalent to A if and only if $\delta=1$; (ii) δ-A is vacuous (i.e., each rule satisfies it) if and only if $\delta=0$; and (iii) for each pair $\delta,\delta'\in[0,1]$ with $\delta<\delta'$, δ'-A implies δ-A. In short, decreasing δ from 1 to 0 weakens A monotonically, eventually to a vacuous requirement. Our parametrizations of *efficiency* and *strategy-proofness* are in line with this spirit, and in order to weaken the axioms monotonically, we use the Hausdorff distance in the Euclidean space.

Specifically, the parametrization of *efficiency* is obtained by the following procedure. Given an economy, normalize to one the Hausdorff distance (induced by the standard Euclidean distance) between the set of *efficient* allocations and the set of feasible allocations.[4] For each $\delta\in[0,1]$, an allocation is δ*-efficient* if the *normalized* distance between the allocation and the set of *efficient* allocations is $1-\delta$. As δ decreases from 1 to 0, the set of δ*-efficient* allocations expands monotonically, eventually coinciding with the set of feasible allocations. A rule is δ*-efficient* if for each economy, it selects a δ*-efficient* allocation.

Next, to illustrate the parametrization of *strategy-proofness*, let $\delta\in[0,1]$. Fix an economy and an agent. Normalize to one the Hausdorff distance between (i) the set of feasible bundles; and (ii) the intersection of the set of feasible bundles and the lower contour set of his true preference relation at the bundle he receives by telling the truth. Then δ*-strategy-proofness* requires that each bundle he can obtain by misrepresenting his preference relation lie within the normalized distance $1-\delta$ of set (ii). As δ decreases from 1 to 0, the set of bundles the agent can receive by lying expands monotonically, and when $\delta=0$, δ*-strategy-proofness* places no restriction.

Our parametrizations enable us to measure the "degree" of incompatibility of *individual rationality*, *efficiency*, and *strategy-proofness*. In light of Hurwicz's (1972) theorem, one may expect that for $\delta\in[0,1)$ sufficiently close to 1, (i) no rule is *individual rational*, δ*-efficient*, and *strategy-proof*; and (ii) no rule is *individual rational*, *efficient*, and δ*-strategy-proof*. But what we show is much stronger than these conjectures. We establish that for the class of two-agent economies defined on a domain containing linear preference relations, for *each* $\delta\in(0,1]$, statements (i) and (ii) above are true (Theorems 1 and 2, respectively). The remaining case $\delta=0$ is an exception. For (i), the no-trade rule, namely the rule that for each economy,

[3] When the preference domain on which a rule is defined becomes smaller, the scope, and hence strength, of *strategy-proofness* decreases. However, even on the smaller domain, the spirit of *strategy-proofness* that no agent ever benefits from lying about his preference relation, remains the same.

[4] Our definition of feasibility requires that the sum of bundles *equals* the sum of endowments. Since we work with strictly monotone preference relations, the latter definition allows us to ignore those uninteresting allocations that waste some of endowments.