Games and Economic Behavior 86 (2014) 165–177

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

A competitive partnership formation process ☆

Tommy Andersson [a], Jens Gudmundsson [a,*], Dolf Talman [b], Zaifu Yang [c]

[a] Department of Economics, Lund University, Box 7082, SE-222 07 Lund, Sweden
[b] CentER, Department of Econometrics and Operations Research, Tilburg University, 5000 LE Tilburg, The Netherlands
[c] Department of Economics and Related Studies, University of York, York YO10 5DD, UK

ARTICLE INFO

Article history:
Received 1 February 2013
Available online 24 March 2014

JEL classification:
C62
C72
D02

Keywords:
Partnership formation
Adjustment process
Equilibrium
Assignment market

ABSTRACT

A group of heterogeneous agents may form partnerships in pairs. All single agents as well as all partnerships generate values. If two agents choose to cooperate, they need to specify how to split their joint value among one another. In equilibrium, which may or may not exist, no agents have incentives to break up any existing partnerships or form new partnerships. This paper proposes a dynamic competitive adjustment process that always either finds an equilibrium or exclusively disproves the existence of any equilibrium in finitely many steps. When an equilibrium exists, partnership and revenue distribution will be automatically and endogenously determined by the process. Moreover, several fundamental properties of the equilibrium solution and the model are derived.

1. Introduction

Partnership is observed in many social and economic relations. This paper considers a group of self-interested agents or firms who individually choose to act alone, or, if it is to their mutual benefit, form partnerships in pairs. Any agent that stays independent generates a value for herself, whereas a cooperating pair must agree upon how to split their jointly generated value. The values together with the corresponding set of agents form the basis of the partnership formation problem. In this competitive environment agents cannot simply choose whom to cooperate with – if they do not offer a sufficiently generous split of the joint value, their potential partner may look elsewhere for agents to cooperate with. Hence, each agent faces a trade-off between trying to maximize her own benefit while still bestowing her partner with a large enough fraction of their joint value. In equilibrium, no agents have incentives to break up any existing partnerships or form

☆ We are very grateful to two anonymous referees and the Associate Editor for their detailed comments and constructive suggestions which helped us to improve the paper considerably. We would also like to thank Vincent Crawford for his encouragement and very helpful comments, and Bettina Klaus and Flip Klijn for their extensive comments, and many participants for their useful feedback at several universities, the 2012 Economic Theory and Applications Workshop in Malaga, the 2012 York Annual Symposium on Game Theory, the XI International Meeting of the Society for Social Choice and Welfare in New Delhi, and the 2013 Economic Design Conference in Lund. T. Andersson wishes to thank the Jan Wallander and Tom Hedelius Foundation (Grant no. P2012-0107:1) and the Ragnar Söderberg Foundation for financial support. J. Gudmundsson wishes to thank the Jan Wallander and Tom Hedelius Foundation (Grant no. P2012-0107:1) for financial support. Z. Yang gratefully acknowledges the financial support by the Netherlands Organisation for Scientific Research (NWO), CentER of Tilburg University, and the University of York.

* Corresponding author.
E-mail addresses: tommy.andersson@nek.lu.se (T. Andersson), jens.gudmundsson@nek.lu.se (J. Gudmundsson), talman@tilburguniversity.edu (D. Talman), zaifu.yang@york.ac.uk (Z. Yang).

http://dx.doi.org/10.1016/j.geb.2014.03.009
0899-8256/

new partnerships. More precisely, no agent gets less than she generates on her own, whereas no two agents get less in total than their joint value.

The partnership formation problem arises from a variety of real life situations. For example, a common practice in software development is pair programming, i.e., when two programmers work together at one workstation. This type of working form falls well within the scope of what can be described as a partnership formation problem. More explicitly, programmers can work alone or in pairs, and the created value (e.g., correctness of the code, designs, etc.) is not only dependent on if the programmers work independently or in pairs, it can also differ depending on the specific partnerships, for instance, due to experience, skills, knowledge, personality, etc.; see e.g., Hannaya et al. (2007). Pair programming is an example of when players are searching for a *complement partner*.[1] In other cases, agents may look for an *opponent partner*. This is, for instance, the case in professional boxing, wrestling, and martial arts where a short-term partnership is formed between two fighters.[2] Also in this case, different partnerships generate diverse values. For example, it is not unreasonable to believe that a fight between the two highest ranked boxers in a specific weight class at a specific boxing federation (IBF, WBF, etc.) generates a higher revenue in terms of ticket sales, sponsorships, etc. than a fight between two lower ranked boxers in the very same weight class and federation. This will undoubtedly also reflect the joint revenue for the boxers. In this case, the value for a fighter without a partner is zero as no revenues can be generated unless she enters the ring. A third example is when an agent searches for a *risk diversification partner*. In this type of partnership, two agents generate their values independently of each other but agree to split the joint value according to some predetermined rule. This was for example the case in the British Dart Organization where Dennis Priestley and Phil Taylor shared their prize money between 1994 and 2000 even when they entered singles darts tournaments.[3]

Two special but prominent cases of the partnership formation problem are the widely studied assignment and marriage matching markets; see Koopmans and Beckmann (1957), Shapley and Shubik (1971), and Becker (1973) among many others. These markets are two-sided in the sense that two disjoint groups are interacting, for example buyers and sellers, firms and workers, or men and women. This separation is erased in the partnership formation problem – agents may well be different (in the values they create), but they are nevertheless all gathered on "the same side" of the market. The assignment markets have been extensively investigated, predominantly with an equilibrium concept that coincides with the core. It is known not only that equilibrium always exists (Koopmans and Beckmann, 1957), but also that the set of equilibria forms a complete lattice (Shapley and Shubik, 1971; Demange and Gale, 1985). In practice, these markets are likely affected by informational asymmetry: the surplus created by a firm employing a worker is information at best available to that particular firm and worker. For assignment markets, several adjustment processes have been proposed that converge to market equilibrium and in addition do not require agents to disclose all of their private information (which they may be reluctant to). Two of the most important ones are due to Crawford and Knoer (1981) and Demange et al. (1986).[4]

Unlike the assignment markets which always have an equilibrium, there exist partnership formation problems that do not have an equilibrium. For instance, consider the following problem with three agents – called 1, 2, and 3. Assume there is no gain from staying independent, whereas any partnership generates a value of 3 dollars. Formally, a matching is used to keep track of who is cooperating with whom, under the restriction that each agent at most may have one partner. Hence, if agents 1 and 2 form a partnership, then agent 3 necessarily remains single. Suppose that agents 1 and 2 allocate one dollar to agent 1 and two dollars to agent 2 out of their jointly generated 3 dollars. This situation is not stable as agent 3 can lure agent 1 away from agent 2 by offering him two dollars – this is beneficial for agent 1 (he gets two instead of one) as well as for agent 3 (he gets one instead of zero). Hence, the partnership between agents 1 and 2 is broken up, and a new one is formed between agents 1 and 3. By symmetry, this situation is also unstable. Applying similar arguments to different divisions of the joint values, one can verify that there exists no equilibrium.

To ensure the existence of equilibrium, several necessary, or sufficient, or both conditions have been identified. Chiappori et al. (2012) and Talman and Yang (2011) use the linear programming approach to examine the existence problem, while Eriksson and Karlander (2001) explore graph-theoretic methods. In particular, Talman and Yang (2011) introduce a general and natural sufficient condition, called the oddness condition (their Assumption 1), which is easily satisfied by the assignment markets. Chiappori et al. (2012) demonstrate that the existence of equilibrium is restored if the economy is duplicated by "cloning" each agent.[5]

[1] This is also common in many professional doubles racket sports (tennis, table tennis, badminton, etc.). For example, a tennis player that is a big server may search for a partner who is a big returner or an aggressive volleyer. In professional doubles racket sports, the joint value is created through the prize money that is generated by forming a specific partnership. If a player cannot find a doubles partner she can stay independent by entering the singles tournament.

[2] There are even centralized "markets" that boxing promoters use to find suitable partners (i.e., opponents). For example, MatchBox Inc. arranged 99% of all women's fights in Europe in 2012 according to their website (www.boxingmatchmaker.com).

[3] In fact, this type of agreement is common in darts. In an interview in The Guardian (January 12, 2006), the spokesman of the British Dart Organization, Robert Holmes, claimed that "It is not unusual for players to share prize money".

[4] All papers cited so far have assumed quasi-linear utility. The problems have been analyzed for more general structures on the utility functions; see for instance Demange and Gale (1985), Kaneko and Yamamoto (1986), Quinzii (1984), and Svensson (1983). In turn, there is an extensive literature on matching markets where monetary transfers are not allowed, ranging from the house swapping market of Shapley and Scarf (1974) to the marriage and roommate problems of Gale and Shapley (1962).

[5] Relatedly, Sotomayor (2005) shows that each core outcome coincides with what we call an equilibrium. Klaus and Nichifor (2010) study properties of equilibrium using a consistency axiom.