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On the convergence to the Nash bargaining solution for action-dependent bargaining protocols

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ABSTRACT

We consider a non-cooperative multilateral bargaining game and study an action-dependent bargaining protocol, that is, the probability with which a player becomes the proposer in a round of bargaining depends on the identity of the player who previously rejected. An important example is the frequently studied rejector-becomes-proposer protocol. We focus on subgame perfect equilibria in stationary strategies which are shown to exist and to be efficient. Equilibrium proposals do not depend on the probability to propose conditional on the rejection by another player. We consider the limit, as the bargaining friction vanishes. In case no player has a positive probability to propose conditional on his rejection, each player receives his utopia payoff conditional on being recognized. Otherwise, equilibrium proposals of all players converge to a weighted Nash bargaining solution, where the weights are determined by the probability to propose conditional on one's own rejection.

1. Introduction

This paper examines the convergence of equilibrium payoffs to the asymmetric Nash bargaining solution in a non-cooperative bargaining game. In contrast to the existing literature on this topic, we allow for the proposer selection process to be action-dependent, that is, influenced by the players' actions throughout the game.

The study of non-cooperative bargaining games has been strongly influenced by Rubinstein (1982). In his seminal paper, the unique division of a surplus among two impatient players is supported by subgame-perfect equilibrium. In unanimity bargaining games with more than two players, Herrero (1985) and Haller (1986) show that the uniqueness of subgame-perfect equilibrium payoffs is not preserved.[4] Therefore, the literature has focused attention on those subgame-perfect equilibria which are in stationary strategies.

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[4] One way to restore the uniqueness of subgame-perfect equilibrium is to deviate from unanimous agreement and consider instead a bargaining process where an agreement is reached in several steps and only a subset of the players bargain with each other at each step. Examples of such "partial agreements"

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The convergence of equilibrium payoffs in the limit as the cost of delay becomes small has been studied for a variety of different bargaining protocols, see Binmore et al. (1986), Hart and Mas-Colell (1996), Laruelle and Valenciano (2008), Miyakawa (2008), Britz et al. (2010), and Kultti and Vartiainen (2010). All those protocols have in common that they are action-independent: The actions taken by the players in the game have no effect on the identity of the next proposer. We argue that the focus on action-independent protocols alone is a serious limitation because action-dependent protocols are very common in other strands of the bargaining literature. One simple and intuitively appealing example is the protocol where the player who rejects the current proposal is automatically called to make the next proposal. This rejector-becomes-proposer protocol has been introduced in Selten (1981) and has been studied extensively in both the bargaining and the coalition formation literature, see for example Chatterjee et al. (1993), Bloch (1996), Ray and Vohra (1999), Imai and Salonen (2000), and Bloch and Diamantoudi (2011).

The protocol we study in this paper is more general than the rejector-becomes-proposer protocol. Following Kawamori (2008), we are interested in the case where the identity of the player who rejects a proposal may influence the probability by which a particular player becomes the next proposer. Since now the accept and reject decisions of the players influence the selection of the proposer, this leads to an action-dependent protocol. Such protocols are considerably more difficult to analyze than action-independent ones, and the literature has identified a number of cases where both types of protocol lead to surprisingly different results. For instance, Chatterjee et al. (1993) provide examples for non-existence of equilibria with immediate agreement in the context of an action-dependent protocol. On the contrary, it has been shown in Okada (1996) that delay cannot occur at equilibrium and in Okada (2011) that equilibria exist when the protocol is action-independent.[5]

Our analysis of stationary subgame perfect equilibria reveals that the set of equilibrium proposals only depends on the bargaining protocol through the probabilities of making counter-offers. A player's probability of making a counter-offer is defined as the probability for that player to become the proposer, given that the previous proposal has been rejected by *this* player.

We show that equilibrium proposals of all players converge to a weighted Nash bargaining solution, where the weights are proportional to the probabilities of making a counter-offer. One exception is the case when the probability of making a counter-offer is zero for all players. In this case the proposer in the initial round obtains his utopia payoff, that is his highest payoff in the set of feasible payoffs that satisfy all the individual rationality constraints. Equilibrium proposals of all players are independent of the continuation probability and do not converge to a common limit.

One implication of our analysis is that the probability of making a counter-offer is a crucial determinant of a player's bargaining power. The existing results on non-cooperative bargaining games are for action-independent protocols only and do not distinguish between the probability of making a proposal and the probability of making a proposal conditional on a rejection. Our paper identifies the latter probabilities as the source of bargaining power.

2. The bargaining game

We consider a bargaining game between finitely many players in the set $N = \{1, \ldots, n\}$. Each player individually can only attain a disagreement payoff which we normalize to zero. However, the players can jointly achieve any payoff vector v in a set $V \subset \mathbb{R}^n$ if they unanimously agree on such a payoff vector. Each player is assumed to be an expected utility maximizer. The set V of feasible payoffs and the bargaining protocol are the main primitives of the model. We now introduce each in turn.

For vectors u and v in $\mathbb{R}^n$, we write $u \geqslant v$ if $u_i \geqslant v_i$ for all $i \in N$, $u > v$ if $u \geqslant v$ and $u \neq v$, and $u \gg v$ if $u_i > v_i$ for all $i \in N$. A point v of V is said to be *Pareto-efficient* if there is no point u in V such that $u > v$. A point v of V is said to be *weakly Pareto-efficient* if there is no point u in V such that $u \gg v$. We write V_+ to denote the set $V \cap \mathbb{R}^n_+$.

Our first assumption is as follows:

[A1] The set V is closed, convex, and comprehensive from below. There is a point $v \in V$ such that $v \gg 0$. The set V_+ is bounded. Each weakly Pareto-efficient point of V_+ is Pareto-efficient.

We denote the set of Pareto-efficient points of V by P and write P_+ for the set $P \cap \mathbb{R}^n_+$. A vector $\eta \in \mathbb{R}^n$ is called a *normal vector* to V at a point $v \in V$ if $(u - v)^\top \eta \leqslant 0$ for all $u \in V$. In addition, a normal vector to V at v is said to be a *unit normal vector* if $\|\eta\| = 1$.

[A2] There is a continuous function $\eta : P_+ \to \mathbb{R}^n$ such that $\eta(v)$ is a unit normal vector to V at the point v.

Assumption A2 implies that the boundary P_+ does not have kinks. Notice that in view of Assumption A1 we have $\eta_i(v) > 0$ for every $i \in N$ such that $v_i > 0$.

can be found in Chae and Yang (1994), Krishna and Serrano (1996), and Suh and Wen (2006). A similar approach has been applied to a coalition formation problem by Moldovanu and Winter (1995).

[5] Duggan (2011) presents a very general coalitional bargaining model where equilibrium existence is shown for action-independent protocols. The paper points out that a similar approach to establish equilibrium existence would not work when the protocol is action-dependent.