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Note

A folk theorem for stochastic games with private almost-perfect monitoring ☆

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ABSTRACT

We prove a folk theorem for stochastic games with private, almost-perfect monitoring and observable states when the limit set of feasible and individually rational payoffs is independent of the state. This asymptotic state independence holds, for example, for irreducible stochastic games. Our result establishes that the sophisticated construction of Hörner and Olszewski (2006) for repeated games can be adapted to stochastic games, reinforcing our conviction that much knowledge and intuition about repeated games carries over to the analysis of irreducible stochastic games.

1. Introduction

The class of stochastic games includes models in which persistent shocks, stock variables representing human or natural resources, technological innovations, or capital play an important role. Stochastic games are extensively used in economics since they capture dynamic interactions in rich and changing environments. It is sometimes natural to assume that players cannot observe others' actions in these dynamic interactions. For example, consider an oligopolistic market in which firms now set prices with their customers bilaterally in each period (Stigler, 1964). The firms cannot observe competitors' price offers, but they obtain some information about these from their own sales that are unobservable to competitors. This is an example of imperfect *private* monitoring where players cannot directly observe others' actions, but receive some private signals, which are imperfect indicators of the action taken in the current period. Such imperfect monitoring is extensively studied in repeated games, which one can view as stochastic games whose state variable is fixed. In this paper, we prove a folk theorem for stochastic games with private almost-perfect monitoring and observable states under the assumption that the limit set of feasible and individually rational payoffs is independent of the state (we call the assumption the *asymptotic state independence*).

In general, when we analyze a repeated game with private monitoring in which there is no available public signal for players to coordinate on, there is no obvious recursive structure available (because players' beliefs about opponents' histories become increasingly complex as time proceeds). To avoid these obstacles, the literature has focused on *belief-free* equilibria,

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introduced by Piccione (2002), and simplified and extended by Ely and Välimäki (2002) and Ely et al. (2005). In this class of equilibria any continuation play is optimal *whatever private histories other players might have observed*, so that players need not form beliefs about opponents' histories in order to compute their optimal behaviors.

For illustration, suppose that John and Susie repeatedly play the following prisoners' dilemma.

	C	D
C	$2, 2$	$-1, 3$
D	$3, -1$	$0, 0$

Note that Susie can ensure that John receives at least 2 when she plays C, and that he receives at most 0 when she plays D. Then John might have an incentive to play C today if Susie is more likely to play C tomorrow when she receives a sufficiently informative signal that John played C today. By suitably choosing the probability with which she plays C tomorrow as a function of her private information today, Susie can ensure that John is indifferent between C and D. In turn, because he is indifferent between C and D, it is optimal for him to condition his play tomorrow on his private information so as to make Susie indifferent between C and D. Strategy profiles of this sort constitute sequential equilibria with the belief-free property. Ely and Välimäki (2002) use this belief-free approach to prove the folk theorem in repeated prisoners' dilemmas. But Ely et al. (2005) show that Ely and Välimäki's construction does not extend to general stage games.

Hörner and Olszewski (2006) (hereafter HO) exploit the essential feature of belief-free equilibrium to prove the folk theorem for general stage games. To do so, they divide the repeated game into a sequence of T-period block games. Provided that T is sufficiently large, the payoff structure of prisoners' dilemma can be recovered from any stage game, using two T-period block strategies for each player: a "good" strategy and a "bad" strategy, which correspond to C and D in the prisoners' dilemma above. John might prefer to play the good strategy if Susie is more likely to play her good strategy in the next block when she observes a sufficiently informative signal that John played his good strategy in the current block. By suitably choosing the probability with which she plays the good strategy in the next block as a function of her current block's private histories, Susie can ensure that John is indifferent between the good and bad strategy at the beginning of each block regardless of what she observed before, and can also ensure the sequential rationality of his strategy during each block. It is the latter requirement that creates additional complications in HO's construction relative to the construction of belief-free equilibrium.

1.1. Our construction

The heart in this paper lies in the adaptation of HO's construction from repeated games to stochastic games. To begin, we divide the entire stochastic game into consecutive T-period block games, for which we define a "good" strategy and a "bad" strategy. Then, we introduce L-period Markov strategies that are repeatedly played within the block. It is well known (Blackwell, 1965) that any extreme point of the set of feasible payoffs in a stochastic game is achieved by a pure Markov strategy profile. Hence, any targeted payoff profile in the set is approximately achieved by repeatedly playing the L-period Markov strategy profile that consists of playing a finite sequence of pure Markov strategy profiles.[1] Although strategies' payoffs in stochastic games can be heavily dependent on the initial state, it is shown that the payoffs generated by the L-period Markov strategies depend little on the initial and final state during the L-period game when the asymptotic state independence holds.[2] Hence, an L-period game in which these L-period Markov strategies are used is an "almost invariant stage game", and the repeated play of these almost invariant stage games can be regarded as "almost repeated game". By using these L-period strategies of the "almost repeated game", we are able to construct "good" and "bad" strategies as in HO. We ensure that these strategies exhibit the belief-free property in the beginning of each block and sequential rationality during each block by defining carefully the transition probabilities between the good and bad strategies as one block ends and the next begins.

Another minor difficulty presented by the stochastic game environment is that altering current actions affects not only the distribution of private signals, but also the distribution of future states. This implies that a player may have an incentive to deviate solely to ensure an advantageous distribution of future states. For example, if, during a play of a strategy profile, a player's continuation payoffs are higher from state ω^{t+1} than from state $\tilde{\omega}^{t+1}$, then in period t the player has an incentive to choose an action that makes ω^{t+1} more likely. To contend with this issue, one can ensure that when an opponent's observations suggest that a player has chosen this kind of advantageous action, the probability that the opponent plays the bad strategy in the next block goes up. By carefully conditioning the transition probabilities on the realized states, we are able to maintain both the belief-free property and the within-block sequential rationality.

The folk theorem for stochastic games has already been proved under both perfect monitoring and imperfect public monitoring. In the case of perfect monitoring, Dutta (1995) proved the folk theorem for stochastic games when the asymptotic

[1] More specifically, any payoff profile v in the set of feasible payoffs can be written as convex combination $\sum_k \lambda_k v_k$ with an extreme point v_k being achieved by a pure Markov strategy profile g_k. Then, by choosing L_k and L such that $L_k/L \approx \lambda_k$ and $L = \sum_k L_k$, consider the following L-period Markov strategy profile: play g_1 for L_1 periods, followed by g_2 for L_2 periods and so on. Now v is approximately attained by repeatedly playing this L-period Markov strategy profile.

[2] The proof follows from some results in Dutta (1995).