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Note

Robust and secure implementation: equivalence theorems

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ABSTRACT

This paper shows that in private value environments, strategy-proofness and the rectangular property are necessary conditions for (full) robust implementation (Bergemann and Morris, 2011). As corollaries, we obtain the equivalence between robust and secure implementation (Saijo et al., 2007), the revelation principle for robust implementation, and characterization of double implementation in robust and secure implementation.

1. Introduction

Implementation theory focuses on mechanisms for decentralized social decision making among agents. The principal designs a game (called *a mechanism*) that the agents play to implement a social goal called a *social choice function* (SCF). We consider the problem of *full implementation* in private value environments: that is, we require that a mechanism have at least one equilibrium with every equilibrium outcome being consistent with the SCF-recommended outcome. In this context, two novel approaches—*secure implementation* and *robust implementation*—have been proposed to refine the existing ideas on implementation.

Secure implementation was proposed by Saijo et al. (2007) to overcome the defect of (weakly) dominant strategy implementation. They argue that even if a mechanism has a dominant strategy profile, there may be a Nash equilibrium whose outcome is different from that of the dominant strategy profile, and the agents may choose it rather than the dominant strategy profile. Therefore, they propose that a mechanism should *securely* implement the SCF, in that SCF is implemented in at least one weakly dominant strategy and all Nash equilibria at the same time. They show that the combination of *strategy-proofness* and *the rectangular property* characterizes securely implementable SCFs, where the rectangular property is newly introduced by Saijo et al. (2007) as a necessary condition for secure implementation. They also provide examples of surplus-maximizing SCFs satisfying strategy-proofness and the rectangular property in some quasi-linear environments. Berga and Moreno (2008), Bochet and Sakai (2010), and Fujinaka and Wakayama (2008, 2010) also study securely implementable SCFs in various other social choice problems.

Robust implementation has a different motivation: the problem of common knowledge assumptions in implementation problems. Bergemann and Morris (2011) first studied robust implementation regarding the problem of full implementation

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by general mechanisms. They argue that agents may not have a common prior on their payoff types, and even otherwise, the principal may be unable to know the prior in practice. Though these are commonly assumed as in Bayesian implementation (Postlewaite and Schmeidler, 1986; Jackson, 1991; Palfrey and Srivastava, 1989), accordingly, Bergemann and Morris (2011) propose that a mechanism should *robustly* implement the SCF, in that SCF is implemented independently of those common knowledge assumptions. They also characterize robust implementable SCFs in general environments including interdependent value environments. Bergemann and Morris (2009b, 2009a) and Artemov et al. (2012) also study other types of robust implementation.

The purpose of this paper is to clarify the relationship between secure and robust implementation in private value environments. Saijo et al. (2007) partially consider this relationship. Adopting a unique version of robust implementation, they show that any securely implementable SCF is robustly implemented by the associated direct mechanism.[2] Thus, secure implementability implies robust implementability. This paper shows that the converse holds: robust implementability implies secure implementability; further, our result holds even under a weaker requirement for robust implementation than that in Saijo et al. (2007).

This result has several important corollaries: equivalence between robust and secure implementability; a characterization of robust implementable SCFs; the revelation principle for robust implementation: and a characterization and the revelation principle for double implementation with robust implementation and secure implementation.

The paper is organized as follows. Section 2 provides the basic notation and introduces the concepts of implementation. Section 3 provides the main results, the corollaries, and a discussion on the relation between robust implementation in this paper and Bergemann and Morris (2011).

2. Setup

Let $I = \{1, 2, \dots, |I|\}$ denote the set of agents. Each agent $i \in I$ has a nonempty and countable set of possible payoff types Θ_i.[3] Let $\Theta = \prod_{i\in I} \Theta_i$. X is the set of social outcomes. Each agent $i \in I$ has a payoff function $u_i : X \times \Theta_i \to \mathbb{R}$, where $u_i(x, \theta_i)$ denotes the payoff of agent i when the outcome is x and his payoff type is θ_i. The payoff is determined independently from the other agents' payoff types; that is, we assume private-value preferences. An SCF f is a mapping from Θ to X.[4] A mechanism $\mathbf{M}$ is defined as a pair of a message space $M = \prod_{i\in I} M_i$ and an outcome function $g : M \to X$; i.e., $\mathbf{M} = (M, g)$. Given an SCF f, its associated *direct mechanism* is the mechanism where the message space is Θ and the outcome function is f; i.e., $\mathbf{M}^D = (\Theta, f)$.

For simplicity, given any profile of sets $(A_i)_{i\in I}$, $(B_i)_{i\in I}$, and functions $(\gamma_i)_{i\in I}$ such that $\gamma_i : A_i \to B_i$ for each $i \in I$, we describe (i) $(\gamma_i)_{i\in I}$ as γ, (ii) $(\gamma_1(a_1), \gamma_2(a_2), \dots, \gamma_{|I|}(a_{|I|}))$ as $\gamma(a)$, and (iii) $(\gamma_1(a_1), \dots, \gamma_{i-1}(a_{i-1}), \gamma_{i+1}(a_{i+1}), \dots, \gamma_{|I|}(a_{|I|}))$ as $\gamma_{-i}(a_{-i})$ for each $a = (a_1, a_2, \dots, a_{|I|}) \in \prod_{i\in I} A_i$.

2.1. Secure implementation

Given a mechanism $\mathbf{M} = (M, g)$ and a payoff type profile $\theta \in \Theta$, we can define the *complete information game*, where all agents' payoff types are common knowledge among the agents. A *Nash equilibrium* $s \in M$ in this game is defined as follows: for each $i \in I$ and each $m_i \in M_i$,

$$u_i\big(g(s), \theta_i\big) \geqslant u_i\big(g(m_i, s_{-i}), \theta_i\big)$$

A message $s_i \in M_i$ on $\mathbf{M}$ is a *(weakly) dominant strategy* for $\theta_i \in \Theta_i$ iff for each $m \in M$,

$$u_i\big(g(s_i, m_{-i}), \theta_i\big) \geqslant u_i\big(g(m), \theta_i\big).$$

Note that the existence of a dominant strategy profile does not deny the possible existence of Nash equilibria with different outcomes. Therefore, Saijo et al. (2007) require that a mechanism should securely implement the SCF in the sense that it implements SCF in some dominant strategy and all Nash equilibria.

Definition 1 *(Secure Implementation).* A mechanism $\mathbf{M} = (M, g)$ *securely implements* an SCF f iff for each $\theta \in \Theta$, (i) there exists a dominant strategy profile $s \in M$ such that $g(s) = f(\theta)$ and (ii) for each Nash equilibrium s', $g(s') = f(\theta)$. An SCF is *securely implementable* iff there exists a mechanism that securely implements it.

[2] As for requirements for independence from common knowledge assumptions, robust implementation in both us and Saijo et al. (2007) is the same as in Bergemann and Morris (2011). The only difference that arises is from the varying definitions of interim implementation: see Section 3.2.

[3] We assume countability of the type set for consistency with Bergemann and Morris (2011). All results hold even under generalized notation as in Saijo et al. (2007).

[4] Considering only SCFs rather than social choice *sets* or *correspondences* is the same as the analysis in both Saijo et al. (2007) and Bergemann and Morris (2011). Since some of our main results are related to the direct mechanisms, which are well-defined only under SCFs, we too take the same approach.