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Strategy-proof contract auctions and the role of ties ☆

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A B S T R A C T

A contract auction establishes a contract between a center and one of the bidders. As contracts may describe many terms, preferences over contracts typically display indifferences. The Qualitative Vickrey Auction (QVA) selects the best contract for the winner that is at least as good for the center as any of the contracts offered by the non-winning players. When each bidder can always offer a contract with higher utility for the center at an arbitrarily small loss of her own utility, the QVA is the only mechanism that is individually rational, strategy-proof, selects stable outcomes, and is Pareto efficient. For general continuous utility functions, a variant of the QVA involving fixed tie-breaking is strategy-proof and also selects stable outcomes. However, there is no mechanism in this setting that in addition also selects Pareto efficient outcomes.

1. Introduction

In a standard auction, bidders compete only on the price they pay (or are paid, in the case of a reverse auction). However, in many situations, bidders also compete on other additional attributes, which could be laid down in a contract. In some settings, the bidders may not even compete on price at all, for instance, if the price is fixed in advance. In a *contract auction* one out of many possible contracts is selected between one player, which we call the *center*, and one winner out of a set of other players, called the bidders. A contract describes all terms of the arrangement between the center and the winner, such as the quality of service, deadlines, reputation, shipping, and payment method, but none of these are obligatory elements. For example, the center may be a company putting out a request for proposals, a governmental organization with a fixed budget acquiring a service from one of the available public transport companies, a hospital hiring one out of many applying doctors, or a home owner requesting a construction bid for a house extension.

In such a contract auction setting we are interested in defining a mechanism that selects an outcome that is *stable*, that is, a contract such that the center cannot get a better deal with another bidder, nor a better deal with the winner without reducing the winner's utility. In particular, we would like to provide bidders with a strategy for this mechanism that is *dominant* (that is, the best the bidder can do irrespective of the other bidders) such that stable outcomes are guaranteed. When the contract consists just of a price for a certain item or service, in the Vickrey or sealed-bid second-price auction

☆ This paper gives a characterization of the mechanism presented at the 10th ACM Conference on Electronic Commerce in 2009 (Harrenstein et al., 2009), and specifically analyzes the role of ties.

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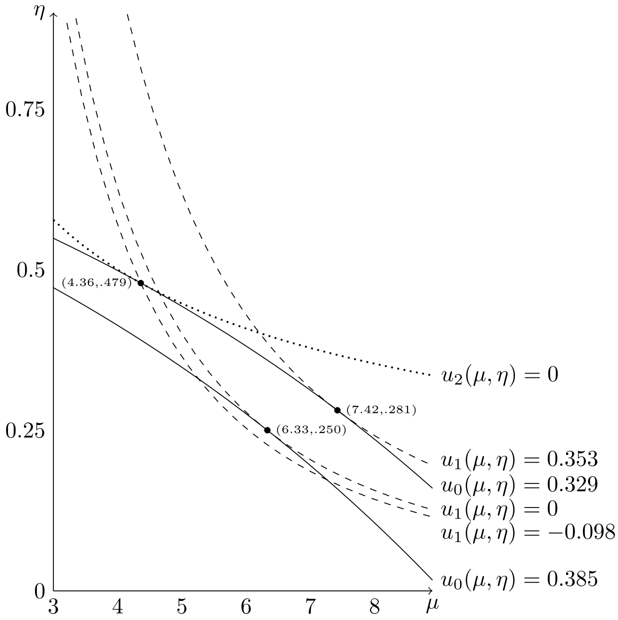

Fig. 1. The situation described in Example 1. The solid curves are indifference curves for the center, the dashed curves are indifference curves for Firm 1, and the dotted curve is an indifference curve for Firm 2.

truthful bidding is a dominant strategy—and thus the Vickrey auction is *strategy-proof*—and selects the highest bidder as a winner (Vickrey, 1961). In this paper we show how to generalize the Vickrey auction to the contract auction setting.

When the set of possible contracts is finite and the preferences of all players over possible contracts do not allow for indifferences, that is, are a linear order, we say the preferences are *strict*. Under this strict preference assumption, a contract auction can be seen as a special case of matching with contracts (Hatfield and Milgrom, 2005) where on one side there is only one player. This illustrates the powerful generality of the matching-with-contracts framework. However, these assumptions of finiteness and strictness of preferences are restrictive and they severely limit application of the framework. For example, the regular Vickrey auction is not a special case, because the possible payments constitute an infinite outcome space. Also, when there are two or more dimensions to a contract—such as both a payment and a delivery date—one would naturally expect indifferences in the center's preferences, if a lower value in the one dimension can be offset by a higher value in the other one. Rather, the center's preferences in such a situation are often lucidly modeled by indifference curves, where the center is indifferent across all the points that lie on the curve. Below we give an example of an application domain that illustrates such a possibly infinite domain of contracts, and preferences that are not strict.

Example 1. Deloxdu University has received a donation to build a new building to house a new center focusing on the intersection of computer science and economics. The donation specifies a fixed budget and requires that the university solicits bids from different firms for the construction of the building, according to an open and clearly specified process. Many attributes of the building, such as the numbers of offices and classrooms, are specified by the university. This leaves only two attributes for bidders to compete on: the energy usage $\eta(> 0)$ of the building and the amount of time $\mu(> 0)$ before the relevant people can move into the building. The firms cannot compete on price because the donation specifies a hard budget for the building that cannot be reallocated. After ample deliberation in committees, the university decides that its utility function is $u_0(\eta, \mu) = .9^{\mu}(1 - \eta)$, reflecting a discount factor of .9 and a long-run utility of occupying the building of $1 - \eta$. This utility function is communicated to the firms the university solicits bids from.

Two firms enter the competition. Firm 1 specializes in energy efficient building, but tends to be slow. Its utility function for winning a contract $\omega = (\eta, \mu)$ is $u_1(\eta, \mu) = 1 - 10\mu^{-2}\eta^{-1}$, which takes into account the resulting revenue for the firm. This reflects that taking either μ or η down to zero will take the cost to infinity and μ more quickly so. Firm 2 specializes in speedy construction, but tends to produce energy inefficient buildings. Its utility function for winning a contract is $u_2(\eta, \mu) = 1 - \mu^{-1}\eta^{-2}$. This reflects that, again, taking either μ or η down to zero will take the cost to infinity and, in this case, η more quickly so.

Naturally, the university would welcome it if each firm were to put forward a proposal that maximizes the university's utility under the constraint that the firm does not have negative utility. Some calculation shows that for Firm 1, the corresponding bid would be $\mu = 6.33$, $\eta = .250$, resulting in $u_0 = .385$ and $u_1 = 0$. For Firm 2, the corresponding bid would be $\mu = 4.36$, $\eta = .479$, resulting in $u_0 = .329$ and $u_2 = 0$. The reader is referred to Fig. 1 for a graphical representation of the situation. Firm 1 has the more desirable proposal: while it will take longer for the building to be ready to occupy, the gains in energy efficiency more than make up for it.