ARTICLE IN PRESS YGAME:2178

Games and Economic Behavior ••• (••••) •••–•••

Contents lists available at SciVerse ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Pricing traffic in a spanning network

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ARTICLE INFO

Article history:
Received 27 June 2011
Available online xxxx

JEL classification:
C71
D63

Keywords:
Connection game
Core stability
Routing-proofness
Minimal cost spanning tree or forest

ABSTRACT

Users need to connect a pair of target nodes in the network. They share the fixed connection costs of the edge. The system manager elicits target pairs from users, builds the cheapest forest meeting all demands, and choose a cost sharing rule satisfying:
Routing-proofness: a user cannot lower his cost by reporting as several users along an alternative path connecting his target nodes;
Stand Alone core stability: no group of users pay more than the cost of a subnetwork meeting all connection needs of the group.
We construct two such rules. When all connecting costs are 0 or 1, one is derived from the random spanning tree weighted by the volume of traffic on each edge; the other is the weighted Shapley value of the Stand Alone cooperative game. Both rules are then extended by the familiar piecewise-linear technique. The former is computable in polynomial time, the latter is not.

1. Introduction

We consider a network between a set of nodes $i, j, \dots,$ serving the traffic demands of a set of users. Each user requests an (undirected) connection between a pair of *target nodes* i and j; several users may request the same connection. The network must connect, directly or indirectly, all pairs of target nodes. There are no variable congestion costs: building a given edge imposes a fixed cost (to cover the construction of some infrastructure, or some access fee) that may depend on the end points of the edge, but not on the amount of traffic on this edge. Users are satisfied with an indirect connection between their target nodes, therefore the efficient network is a minimal cost forest (disjoint union of trees) *spanning* all the traffic demands (connecting all target pairs).

A familiar special case is the minimal cost spanning tree problem (thereafter MCST: Claus and Kleitman, 1973; Bird, 1976) where users are attached to a node and each user needs to connect to a special node (the *source*). With n nodes, the $n-1$ types of users in the MCST problem become $\frac{n(n-1)}{2}$ types in our model. To the familiar applications of the MCST model to multicast transmission (Hoefer, 2013), our model adds the special case of the network synthesis problem where links are congestion-free (as in the connection game of Anshelevich et al., 2004, and the capacity synthesis problem of Bogomolnaia et al., 2010).

Given the complex externalities in this network design problem, the challenge is to propose a compelling division of the efficient cost among the participants. For over three decades, the most fruitful approach in the MCST and other combinatorial optimization problems[1] has been cooperative game theory, and the key concept Stand Alone core stability: no group of users should be charged more than the cost of a subnetwork meeting the needs of the group members (Bird, 1976;

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[1] Such as the traveling salesman, facility location and flow synthesis problems: see Tamir (1991), Granot and Maschler (1998), and Sharkey (1995) for a survey.

0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2013.06.002

Please cite this article in press as: Moulin, H. Pricing traffic in a spanning network. Games Econ. Behav. (2013), http://dx.doi.org/10.1016/j.geb.2013.06.002

Megiddo, 1978; Granot and Huberman, 1984). Adding to core stability natural requirements of continuity and monotonicity with respect to costs (Dutta and Kar, 2004) led more recently to the definition of a certain cost sharing rule, invented independently by several authors (Feltkamp et al., 1994; Norde et al., 2001; Bergantiños and Vidal-Puga, 2007), and dubbed the "folk" solution in Bogomolnaia and Moulin (2010). It applies the Shapley value to a canonical reduction of the cost matrix, ensuring the concavity of the Stand Alone cooperative game. The two cost sharing rules we propose reduce to the folk solution in the MCST case.[2]

We introduce a novel type of strategic maneuver of a non-cooperative nature, quite different than the (real or virtual) coordinated secessions prevented by Stand Alone core stability. To implement a personalized price to users, as required by core stability, the system manager must be able to identify the target nodes of every user, for instance by "tagging" individual messages. However in many large networks the users can easily create multiple aliases without being detected, and in this sense they remain anonymous. Gaming a mechanism by using multiple aliases has been discussed in a variety of contexts, including the ranking of web pages (Douceur, 2002; Cheng and Friedman, 2005), scheduling (Moulin, 2007), and voting (Conitzer, 2008). In our model it means that a user with target nodes i, j may be able to lower his charge by reporting as *two* users, one with target nodes i, k, the other k, j, or more generally by reporting as several users, each requesting one edge of a path $i, k_1, k_2, \ldots, k_t, j$ from i to j (we give simple examples at the beginning of Section 2). We call a mechanism *routing-proof* if it is not vulnerable to such maneuvers.[3]

Routing maneuvers distort the report of actual traffic and may create strategic instability, including the possibility of conflicting reporting equilibria (see the examples in Section 2). Because it eliminates gaming opportunities in the traffic reports, routing-proofness is as desirable a property as the familiar strategy-proofness in other resource allocation mechanisms.

Summary of results Recall that for some traffic patterns and costs, there is no core division of the costs. This is already the case in the MCST model: Megiddo (1978), Tamir (1991). The difficulty comes from *Steiner* nodes and edges, namely edges $e = ij$ such that no user has i, j for target pair, and nodes i such that no user has i for one of her target nodes, while they can be used in the optimal spanning forest.[4]

Our results are twofold. We identify two families of connection games where we show that the Stand Alone core is non-empty. Next we construct two cost sharing rules that are core stable and routing-proof in each family (Theorems 1 and 2). The first family (Section 4) places no restriction on the traffic pattern, but restricts all connecting costs c_e to be either 0 or 1. Although coalitions can use Steiner nodes or edges, the Stand Alone core is non-empty. The second family (Section 5) allows for arbitrary connection costs but requires that the traffic be *spanning*, namely the graph of demanded edges is connected and reaches all nodes. Thus every node is demanded, but coalitions may use edges that no one demands. An example is the standard MCST model, where all nodes other than the source are occupied by at least one agent.

Our rules charge by design the same price to all users with the same target pair.[5] A consequence of routing-proofness is that cost shares are sensitive to usage (although actual costs are not). E.g., both rules satisfy the following property: if m users have target nodes i, j, and one more user with the same demand shows up, the *total* charge to the users with this demand weakly increases, whereas the charge to *any initial* user, whether he had the target nodes i, j or not, weakly decreases.

Both cost sharing rules coincide with the folk solution in the special case of the MCST model; they share with that solution two powerful monotonicity properties: one was mentioned in the previous paragraph; the other is that *all* cost shares increase weakly when *any* connection cost increases (Bergantiños and Vidal-Puga, 2007). Both rules are easy to define (though only one is easy to compute) when connecting costs are all 0 or 1, and are extended to arbitrary costs spanning traffic by means of the *piecewise-linear* technique pioneered in Feltkamp et al. (1994), Norde et al. (2001) and developed in Bogomolnaia and Moulin (2010), Bogomolnaia et al. (2010).

More related literature An early axiomatic model in connection networks (Herzog et al., 1997), applies the Shapley value to share the cost of a multicast tree. A more direct inspiration for this paper is the recent work on *selfish routing* in networks, in particular the *global connection games* introduced in Anshelevich et al. (2004, 2008), where the network technology is the same as here, each user chooses non-cooperatively a route (a sequence of edges connecting their two target nodes), and the cost of each edge is divided equally between all users who choose a path containing this edge (while in our model all messages are routed along the efficient spanning forest). It turns out that a Nash equilibrium always exists (Anshelevich et al., 2004, 2008), that strong equilibrium may or may not exist (Epstein et al., 2009; Hoefer, 2013), and in equilibrium the optimal spanning forest may or may not be built. Remarkably, the relative excess cost of the equilibrium forest can often be bounded a priori (Anshelevich et al., 2004, 2008, see also Nisan et al., 2007, Chapter 19).[6]

[2] See Remark 1 at the end of Section 5.

[3] Note that routing maneuvers are not feasible in the MCST problem, where it is common knowledge that all users want to be connected to the source.

[4] A familiar way to recover core stability is to restrict the power of (proper) coalitions by forbidding them to use Steiner nodes or edges when formulating an objection. Only the grand coalition is allowed such opportunity. This assumption makes little sense in our model, where users can fake a traffic demand involving any node or edge.

[5] For routing maneuvers to be relevant, agents must be able to create aliases without being detected. This implies that the mechanism cannot distinguish identical demands.

[6] The most popular model of selfish routing has *congestion-prone* traffic: if n users route through an edge each will experience a (typically non-monetary) cost $c(n)$ where c is increasing. See Roughgarden (2005) for a survey.

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