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Unpredictability of complex (pure) strategies

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ABSTRACT

Unpredictable behavior is central to optimal play in many strategic situations because predictable patterns leave players vulnerable to exploitation. A theory of unpredictable behavior based on differential complexity constraints is presented in the context of repeated two-person zero-sum games. Each player's complexity constraint is represented by an endowed oracle and a strategy is feasible if it can be implemented with an oracle machine using that oracle. When one player's oracle is sufficiently more complex than the other player's, an equilibrium exists with one player fully exploiting the other. If each player has an incompressible sequence (relative to the opponent's oracle) according to Kolmogorov complexity, an equilibrium exists in which equilibrium payoffs are equal to those of the stage game and all equilibrium strategies are unpredictable. A full characterization of history-independent equilibrium strategies is also obtained.

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1. Introduction

Unpredictable behavior is central to optimal play in many strategic situations, especially in social interactions with conflicts of interest. There are many illustrative examples from competitive sports, such as the direction of tennis serves and penalty kicks in soccer. Other relevant examples include secrecy in military affairs, bluffing behavior in poker, and tax auditing. In these situations, von Neumann and Morgenstern (1944) suggest that, even against a moderately intelligent opponent, a player should aim at being unpredictable. Schelling (1980) argues that the essence of such unpredictable behavior is to avoid simple regularities, which may allow the opponent to predict one's behavior for exploitation. These arguments seem to be based on an intuitive connection between complexity and unpredictability, and this paper formalizes this connection in the context of a repeated two-person zero-sum game. Specifically, I propose a new model of strategic complexity in which unpredictable behavior emerges in equilibrium due to complexity considerations.

The proposed framework has two basic ingredients. First, each player i is endowed with an infinite binary sequence, θ^i, which represents the "sources" of player i's complicated or unpredictable behavior. Second, each player i has to choose an *oracle machine* to implement his strategy, using θ^i as the oracle. Except for being able to ask queries and obtain answers from the oracle, an oracle machine behaves exactly the same as a Turing machine. In particular, each oracle machine has only finitely many instructions, and any successful computation has to terminate within finitely many steps. Thus, the set of strategies that player i can implement is a countable set. Moreover, the oracle θ^i also represents the complexity of player i's strategies. An oracle θ^j is more complex than another oracle θ^i if there is an oracle machine that computes θ^i using θ^j

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as the oracle. On the one hand, if θ^i is computable from θ^j, then any of player i's strategies is simple in the sense that it can be simulated by player j. On the other hand, if θ^i is not computable from θ^j and *vice versa*, each player can implement some strategy that is complex relative to the other player's complexity constraint.

The existence of mutually uncomputable oracles distinguishes my approach from most of the previous literature on repeated games with complexity constraints. Many papers in the literature model strategic complexity using finite automata and use the number of states to measure complexity. In the context of repeated zero-sum games, Ben-Porath (1993) obtains a precise bound on the difference between the sizes of the automata available to the two players so that one can fully exploit the other. Intuitively, when one player can use sufficiently larger automata than the other, the weaker player becomes perfectly predictable. However, to obtain an equilibrium whose value equals to that of the stage game and hence with no player exploiting the other, randomization is necessary and the sizes of the automata available to the two players have to be close to each other.

In contrast, no randomization is allowed in my framework, and the oracles are the only sources of unpredictability. While a randomized strategy is assumed to be unpredictable for any player (even for the one who is using it), in my approach unpredictability is a *relative* notion and is connected to complexity. A strategy can be unpredictable for one player but perfectly predictable for the other. This feature brings about new results that show an intimate relationship between complexity and unpredictability in the context of repeated zero-sum games. It also formalizes the intuition that equilibrium only requires players' behavior to be unpredictable to its opponents. The main results highlight a precise unpredictability requirement in terms of both complexity and statistical patterns for a strategy to be optimal.

This paper contains three main results, which can be classified according to the value obtained in equilibrium. The first result (Theorem 3.1) is concerned with the case where one player fully exploit the other, which resembles previous results in the literature. I show that an equilibrium exists in which one player can fully exploit the other when the former player's oracle is sufficiently more complex than the other player's. The second and the third results are concerned with obtaining a value equal to the value of the stage-game mixed equilibrium. To obtain such a value, the two oracles are necessarily mutually uncomputable (Proposition 3.1), and any of the equilibrium strategies is uncomputable from the other player's perspective. This result, while intuitive, is novel because it shows the necessity of strategic complexity for optimality, and suggests that the role of randomization in the previous works is to provide strategic complexity that cannot be modeled by finite automata.

More specifically, the second result (Theorem 3.2) gives a sufficient condition, called mutual complexity, on the two oracles for equilibrium existence with the stage-game mixed-equilibrium value. Mutual complexity is a stronger requirement than mutual uncomputability, and it requires the two players' oracles to be sufficiently complex relative to each other according to Kolmogorov complexity. This result extends the well-known connection between Kolmogorov complexity and unpredictability in the algorithmic randomness literature (see Downey et al., 2006 for a survey) to a game-theoretical setting. It shows that complexity can be a source for unpredictability in repeated games. The third result (Theorem 3.3) gives a full characterization of equilibrium history-independent strategies under mutual complexity. As mentioned earlier, any equilibrium strategy has to be uncomputable from the other player's perspective. The characterization result shows that optimality also requires unpredictable properties in terms of statistical patterns. Thus, for a sequence of plays to be an equilibrium strategy, it has to not only be complex relative to the other player's oracle but also satisfy frequency requirements.

1.1. Related literature

Conceptually, the model here is close to repeated-game models with finite automata initiated by Aumann (1981), as well as models with bounded recall (Lehrer, 1988). My first result (Theorem 3.1) is similar to Theorem 1 in Ben-Porath (1993), and to many others which consider two-person repeated zero-sum games and show that the "smarter" player exploits his opponent in equilibrium.[2] The main difference between my framework and the existing literature, however, is manifested in my second result (Theorem 3.2) and third result (Theorem 3.3). Without invoking randomization (although, as discussed in Section 4, my results are robust to the introduction of randomization), my results show that unpredictable behavior may emerge in equilibrium due to complexity constraints, but only when both players are "smart" relative to each other in the sense that each player has a strategy that the other player cannot simulate. Moreover, they give a precise requirement of unpredictability in terms of both complexity and statistical patterns (without the introduction of random variables, however). This mutual "smartness" requirement is not present in the existing literature, but my results suggest that it is the key to understand the connection between complexity and unpredictability in repeated games.

There are also models that use Turing machines to measure strategic complexity, including Anderlini and Sabourian (1995) and Hu and Shmaya (2013). Anderlini and Sabourian (1995) consider games with common interests in which only computable strategies are allowed and hence have a very different focus. Hu and Shmaya (2013) also employ oracle machines to model forecasting strategies in the context of expert testing, but the focus there is on when the expert can "outsmart" the test but not on the connection between complexity and unpredictability.

[2] Neyman (1998) obtains asymptotic results, including the exploitation result, in finitely repeated games with finite automata. There are many other papers considering settings with an unconstrained player and a restricted player. Neyman and Okada (1999, 2000) study the value of the repeated game as a function of the constraint on the restricted player's strategic entropy. Gossner and Vieille (2002) consider a situation where one player can only condition his actions on a biased coin and show that bounded strategic entropy emerges endogenously.

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