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# Expectation formation rules and the core of partition function games <sup>☆</sup>

Francis Bloch <sup>a,\*</sup>, Anne van den Nouweland <sup>b</sup><sup>a</sup> Department of Economics, Université Paris I and Paris School of Economics, 106-112 Boulevard de l'Hopital, 75647 Paris Cedex 13, France<sup>b</sup> Department of Economics 435 PLC, 1285 University of Oregon, Eugene, OR 97403-1285, United States of America

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## ABSTRACT

This paper proposes axiomatic foundations of expectation formation rules, by which deviating players anticipate the reaction of external players in a partition function game. The projection rule is the only rule satisfying subset consistency and responsiveness to the original partition of non-deviating players. It is also the only rule satisfying subset consistency, independence of the original partition of deviating players, and coherence of expectations. Exogenous rules are the only rules satisfying subset consistency and independence of the original partition, and the pessimistic rule is the only rule generating superadditive coalitional games.

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## 1. Introduction

The objective of this paper is to provide axiomatic foundations for extensions of the core to games in partition function form. It is well known that, if one moves beyond the highly competitive, zero-sum game environment of [Von Neumann and Morgenstern \(1944\)](#), the worth of a coalition cannot be defined independently of the coalition structure formed by other players. The natural description of a cooperative environment is then a game in partition function form ([Thrall and Lucas, 1963](#)) specifying for each coalition structure and each coalition embedded in that coalition structure, the worth that the coalition can achieve. [Ray \(2007\)](#) contains a thorough discussion of the difference between partition function games and coalitional games, and references to the early literature on partition functions.

Unfortunately, in games in partition function form, the dominance relation which supports the core cannot be defined unambiguously. When a coalition of players deviates, the payoff they expect to obtain depends on the way they expect external players to react to the deviation. This ambiguity has long been recognized – at least since [Aumann \(1967\)](#) – and various definitions of the core have been proposed corresponding to different specifications of the expectations of deviating players on the reaction of external players. For example, [Hart and Kurz \(1983\)](#) describe the  $\alpha$  and  $\beta$  cores, based on pessimistic beliefs where players expect external players to organize in such a way that they minimize the payoffs of deviating players, and the  $\gamma$  and  $\delta$  cores, where players anticipate that coalitions which have been left by some members

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\* Corresponding author.

E-mail addresses: [francis.bloch@univ-paris1.fr](mailto:francis.bloch@univ-paris1.fr) (F. Bloch), [annev@uoregon.edu](mailto:annev@uoregon.edu) (A. van den Nouweland).

of the deviating group either disintegrate into singletons, or stick together.<sup>1</sup> Chander and Tulkens (1997) and de Clippel and Serrano (2008) focus attention on a model where deviating coalitions expect all other players to remain singletons whereas Maskin (2003) and McQuillin (2009) suppose that they expect all other players to form the complement. Shenoy (1979) assumes that deviating players are optimistic and anticipate that external players organize in order to maximize the deviating players' payoffs. Hafalir (2007) compares different core notions based on different expectation formation rules and proves that for convex partition function form games and some expectation formation rules, the resulting cores are nonempty.

Definitions of the core of partition function games proposed in the literature are thus based on ad hoc assumptions on the reaction of external players to the deviation. By contrast, our objective in this paper is to ground the expectations of deviating players on axioms, and derive the core of a partition function game on the basis of properties satisfied by the expectation formation rule. We first propose a set of axioms that pertains to the relation between the current partition and the expectations formed by deviating players. An expectation formation rule is independent of the original partition or independent of the position of deviating players in the original partition if players do not tie their expectations to the current state. It is instead responsive if different partitions of external players always give rise to different expectations. The second set of axioms deals with the compatibility of expectations among groups of players. Compatibility is needed to guarantee that group expectations are well defined. Path independence guarantees that group expectations are independent of the order in which individual expectations are aggregated. Subset consistency prevents disagreement among players over the group expectations. Coherence of expectations introduces a compatibility condition between the expectations formed by  $S$  and its complement. It guarantees that groups hold rational expectations over the behavior of their complements. Finally, we define superadditivity as the property that the coalitional function generated by the expectation formation rule is always superadditive.

We analyze which of the commonly used expectation formation rules satisfy these axioms, and characterize the *projection rule* (by which players anticipate that external players form a coalition structure which is the projection of the current coalition structure) – also known as the  $\delta$  rule in Hart and Kurz (1983) – as the only expectation formation rule that satisfies the two properties responsiveness and subset consistency, or the three properties subset consistency, independence of the position of deviating players in the original partition, and coherence of expectations. If instead of responsiveness to the current partition, we require independence of the current partition, the only rules that satisfy subset consistency are *exogenous rules* where deviating players anticipate external players to organize according to the projection of an exogenous partition  $\mathcal{M}$ . Notice in particular that if  $\mathcal{M}$  is a partition of singletons, the  $\mathcal{M}$ -exogenous rule corresponds to the  $\gamma$  rule of Chander and Tulkens (1997) or the externality-free rule of de Clippel and Serrano (2008), whereas if  $\mathcal{M}$  is the partition formed by the grand coalition, the  $\mathcal{M}$ -exogenous rule specifies that agents anticipate external players to form a single component as in Maskin (2003) and McQuillin (2009). We also note that the *pessimistic rule* (the  $\alpha$  rule) is the only expectation formation rule that satisfies superadditivity. Our final result shows that the expectation formation rule under which balancedness of the coalitional game is equivalent to nonemptiness of the core is the *optimistic rule*.

To the best of our knowledge, our paper represents the first attempt to axiomatize the reaction of external players to a deviation in order to define the core of partition function games. However, the need to specify the partition of external players also appears in studies of extensions of the Shapley value to partition function games. Starting with Myerson (1977), several extensions of the Shapley value to partition function games have been proposed. Recently, Macho-Stadler et al. (2007) have proposed an axiomatization based on the classical axioms of Shapley. De Clippel and Serrano (2008) base their value on axioms of marginality and monotonicity. McQuillin (2009) uses an approach based on the recursion axiom, which states that the solution applied to the game generated by the solution itself should return the same outcome. Grabisch and Funaki (2012) propose an extension of the Shapley value based on the process of coalition formation. Borm et al. (2013) base their extension of the Shapley value on a noncooperative implementation mechanism. Dutta et al. (2010) extend the axioms of consistency and the potential approach to partition function games. While the axioms we discuss in the current paper are applied to a different object than the axioms studied in the context of the Shapley value, there are clear similarities between our approaches. In order to use the potential approach, Dutta et al. (2010) need to define restrictions of partition function games after one player leaves. They propose axioms on restriction operators, including a path independence axiom which guarantees that the restricted games do not depend on the order in which players leave. Implicitly, their axioms embody conditions on the partition formed after a player leaves. By contrast, our axioms apply directly to expectation formation rules. Hence, their axiomatizations and ours are complementary.

The rest of the paper is organized as follows. We present our model of partition function games and expectation formation rules in the next section. Section 3 is devoted to the description of axioms on expectation formation rules. Section 4 contains the axiomatizations of the projection and exogenous rules and a discussion of superadditivity. We discuss the construction of the core of partition function games generated by expectation formation rules in Section 5. Section 6 concludes and proposes directions for future research.

<sup>1</sup> The  $\gamma$  model finds its roots in Von Neumann and Morgenstern (1944) who discuss a game of coalition formation among three agents which requires unanimity and is equivalent to the  $\gamma$  game.

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