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Spectrum value for coalitional games ☆

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ABSTRACT

Assuming a 'spectrum' or ordering of the players of a coalitional game, as in a political spectrum in a parliamentary situation, we consider a variation of the Shapley value in which coalitions may only be formed if they are connected with respect to the spectrum. This results in a naturally asymmetric power index in which positioning along the spectrum is critical. We present both a characterization of this value by means of properties and combinatoric formulae for calculating it. In simple majority games, the greatest power accrues to 'moderate' players who are located neither at the extremes of the spectrum nor in its center. In supermajority games, power increasingly accrues towards the extremes, and in unanimity games all power is held by the players at the extreme of the spectrum.

1. Introduction

The Shapley value (Shapley, 1953) has for decades been one of the main indices used in the literature for measuring the relative power of players in coalitional game situations. The Shapley–Shubik power index (Shapley and Shubik, 1954), the restriction of the Shapley value to simple games, has in particular found wide application for studying voting situations. A voting situation is characterized by the set of agents participating in it and the subsets of this set that have enough power to pass a bill. These two elements together define a simple game.

A comprehensive overview on simple games and power indices can be found in Felsenthal and Machover (1998). For just two examples of the extent to which the Shapley–Shubik and related power indices have been used to measure the power of the agents in major institutions around the world, see Bilbao et al. (2002), which studies the implications of the enlargement of the European Union, and Alonso-Meijide and Bowles (2005), which studies the distribution of power in the International Monetary Fund.

In practice, however, in many political situations the Shapley–Shubik index and its variants have often failed to capture what one would consider a realistic power measure. We put forward here the claim that this is because many papers on the subject do not take into account the relative ideologies of the players, which is of key importance in political situations.

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Consider for example a parliamentary situation in which there are n players, with the same number of votes, and a simple majority of them is required to form a government. A straightforward application of the Shapley value grants each player $1/n$, using symmetry considerations. In real-life parliaments, however, it is intuitively clear to all observers that not all members have equal power. It is highly unusual to see, for example an extreme right party joining an extreme left party in a coalition without any center parties also included in the coalition to bridge political differences between them.

As the above discussion indicates, part of the problem is that the standard Shapley approach assumes that all possible permutations of the players be used in forming coalitions. That means that even highly unlikely coalitions, such as those formed by an extreme left party joining with an extreme right party while by-passing all the parties in between, including their most natural political allies, must be counted equally along with every other coalition.

Different approaches have been proposed in the literature to study situation in which not all coalitions are feasible or equally likely. In many papers, the problem is tackled by considering some structure on the set of players to describe the way in which players can form coalitions. Coalitional games together with these kind of structures are usually denoted games with restricted cooperation.

One of the most widely-studied model of games with restricted cooperation is the restricted communication model proposed by Myerson (1977). In Myerson's approach, in addition to the game itself one considers an undirected graph that describes communication possibilities between the players. A modification of the Shapley value is then proposed under the assumption that coalitions that are not connected in this graph are split into connected components. In contrast, in our approach we focus on permutations, that is, on the way in which coalitions are formed, instead of imposing restrictions directly on possible coalitions.

We propose here an intuitive way to modify the Shapley value by taking the political spectrum explicitly into account. The incorporation of the ideological positions of the agents for the study of the power distribution of a decision-making body was first introduced by Owen (1972). In that work, agents' political positions are given as points in a high-dimensional Euclidean space, and a probability distribution on the set of all permutations is inferred from them. Then, a modification of the Shapley value is proposed based on two properties, namely that an ordering and its reverse ordering should have the same probability and that the removal of a subset of agents should not affect the probabilities assigned to the relative orderings of the remaining agents.

Shapley (1977) proposed taking into account the political positions of the agents as well, using this to develop an asymmetric generalization of the Shapley value. This modification of the original Shapley value was also considered in Owen and Shapley (1989) to study the optimal ideological position of candidates. More recently, Alonso-Meijide et al. (2011) introduced what they termed the distance index. This value for simple games is another modification of the Shapley value that takes into account the ideological positions of agents. Based on Euclidean distances between agents, a probability distribution is constructed that gives high probability to coalitions whose total distance is relatively small.

Even though it is based on ideas similar to the above-cited works, our approach is much simpler. Firstly, we consider only ordinal positions in a one-dimensional space, without further exogenous specification of distances. Secondly, we assign equal probability to all the permutations that are admissible in our model. This simplicity allows for a characterization of the value by means of a set of properties and eases computation of the value. With regard to the properties of the Owen (1972) value, the value introduced here shares the first of those properties but not the second one.

In this work we assume that there exists a spectrum, from 'left to right' according to which the players are ordered linearly. We then impose the condition that as coalitions are formed *à la* Shapley, they must be connected with respect to the spectrum. Hence, we propose a novel way to generalize the Shapley value to games with restricted cooperation in which the restrictions arise from the position of the agents in a one-dimensional spectrum. This leads to an interesting new value that may shed light on relative power measures in situations in which there is a natural ordering of the players.

Perhaps the paper with the most similar general motivation to ours is Gilles and Owen (1999), in which an exogenously given hierarchy amongst players is assumed (as opposed to the exogenously given spectrum as in our paper). A player in the Gilles and Owen (1999) model may join a coalition only if s/he received permission from one or more 'supervisors'. As in this paper, this has the result of limiting the admissible coalitions that may be formed, thus affecting the value. The value in Gilles and Owen (1999), however, differs from the spectrum value because of the different assumptions regarding which coalitions are admissible. In particular, there is no way to define a clear hierarchy in the model of this paper; for any pair of players i and j, there are admissible coalitions that i can join before j joins (hence i cannot depend on 'permission' from j) and admissible coalitions that j can join before i joins.

Nowak and Radzik (1994) introduce a value called the solidarity value by adding a new postulate to the Shapley properties based on the average marginal contributions of the members of coalitions that may be formed. In its basic interpretation, if a coalition S is formed then the players who contribute more to S than the average marginal contribution members of S contribute to supporting the 'weaker' partners in S. Consideration of the solidarity value, however, does not involve any restriction on the admissible coalitions that can be formed, in contrast with the model used in this paper.

Calvo and Gutierrez (2013) start from similar suppositions to those in Owen (1977), namely that coalitional games are endowed with a coalitional structure, an exogenously given partition of the players. When coalitions are formed, the players interact at two levels: first, bargaining takes place among the unions and then bargaining takes place inside each union. Calvo and Gutierrez (2013) make use of the solidarity value in this model: first, unions play a quotient game among themselves according to the Shapley value, and then the outcome obtained by the union is shared among its members by