Games and Economic Behavior 82 (2013) 157–168

Contents lists available at SciVerse ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

A characterization of a family of rules for the adjudication of conflicting claims

William Thomson [1]

Department of Economics, University of Rochester, Harkness Hall, Rochester, NY 14627, United States

ARTICLE INFO

Article history:
Received 11 May 2006
Available online 29 July 2013

JEL classification:
C79
D63
D74

Keywords:
Claims problems
Consistent extensions
Proportional rule
CEA rule
CEL rule

ABSTRACT

We consider the problem of adjudicating conflicting claims, and characterize the family of rules satisfying four standard invariance requirements, homogeneity, two composition properties, and consistency. It takes as point of departure the characterization of the family of two-claimant rules satisfying the first three requirements, and describes the restrictions imposed by consistency on this family and the further implications of this requirement for problems with three or more claimants. The proof, which is an alternative to Moulin's original proof [Moulin, H., 2000. Priority rules and other asymmetric rationing methods. Econometrica 68, 643–684], is based on a general method of constructing consistent extensions of two-claimant rules [Thomson, W., 2007. On the existence of consistent rules to adjudicate conflicting claims: a constructive geometric approach. Rev. Econ. Design 11, 225–251], which exploits geometric properties of paths of awards, seen in their entirety.

1. Introduction

We consider the problem of dividing a social endowment of an infinitely divisible and homogeneous resource among agents having claims on it that cannot be jointly honored. A primary example is when the liquidation value of a bankrupt firm has to be distributed among its creditors. A "division rule" is a function that associates with each such "claims problem" a division of the endowment, which we call an "awards vector" for the problem. This division is interpreted as the choice that a judge or an arbitrator could make. The formal literature devoted to the analysis of this sort of situations originates in O'Neill (1982).[2]

We offer here a geometric and straightforward proof of the characterization, due to Moulin (2000), of the family of rules satisfying four standard invariance requirements. We aim to make this important theorem more easily accessible and to illustrate a technique that has provided answers to an entire class of similar questions. This technique is presented in a didactic way and applied to several examples in Thomson (2007).

The requirements on a rule are the following. First, if claims and endowment are multiplied by the same positive number, then so should the recommended awards. Second, if the endowment decreases from some initial value, the awards vector chosen for the final endowment should be equivalently obtainable in two ways: either the awards vector initially chosen is ignored and the rule is reapplied to divide the final endowment; or this initial awards vector is used as claims vector in dividing the final endowment. Third is a counterpart of this condition pertaining to possible increases in the endowment.

E-mail address: wth2@mail.rochester.edu.

[1] This work was supported by NSF under grants SBR-9731431 and SES-0214691. I thank Diego Dominguez, Eun Jeong Heo, Yoichi Kasajima, Çağatay Kayı, Toyotaka Sakai, Rodrigo Velez, Cori Vilella, Chun-Hsien Yeh, two referees, and an associate editor for their comments.

[2] For surveys, see Thomson (2003), Thomson (2013), Thomson (forthcoming).

0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2013.07.002

Fourth is the variable-population requirement of "consistency": the awards vector chosen for each problem should be in agreement with the awards vector chosen for each "reduced problem" obtained by imagining that an arbitrary subpopulation of claimants leave the scene with their awards, and re-evaluating the situation at that point. For motivation, we refer the reader to Thomson (2003) and the literature cited therein.

Note that no symmetry or anonymity requirement is imposed. Although such requirements are often natural, it is desirable in many other situations to have the option of treating agents differently even when their claims are equal. This allows taking into account other characteristics they have such as income, family responsibility, health status, record of service, and so on.

A rule satisfying the four requirements is defined as follows. First, a "reference ordered partition" of the set of "potential" claimants into "priority classes" is specified.[3] For each problem, one calculates the ordered partition of the set of claimants actually involved that is induced by the reference partition. The components of the induced partition are "handled" in that order: one fully satisfies the claims of all agents in a class before assigning anything to any class with a lower priority. This is common practice (for instance, in bankruptcy court, secured claims have priority over unsecured claims). More interesting is how each class is handled when its turn comes if at that point there is not enough left to satisfy the claims of its members. There are two main cases. If the class coincides with a two-claimant component of the reference partition, a member of a rich family of rules that "link" the proportional, constrained equal awards (CEA), and constrained equal losses (CEL) rules, is applied. If the class has more than two claimants, one of the following rules is applied: the proportional rule, a weighted CEA rule, or a weighted CEL rule. In each of the last two cases, the weights are proportional to a vector of positive "reference weights" assigned once and for all (before the problem that is to be solved is given) to the members of the component of the reference partition of which this class is a subset.

The proof is in two steps. First, the family of two-claimant rules satisfying the three fixed-population axioms is characterized. Second, the family of rules defined over the entire domain and satisfying consistency in addition is identified. We focus on that second, more delicate step, exploiting a technique to pass from two claimants to more than two claimants whose usefulness extends much beyond the question addressed here. Indeed, it can be used very generally to (i) demonstrate the existence of a "consistent extension" to general populations of an *a priori* given two-claimant rule if such an extension exists, and to identify this extension, or (ii) to show that no such extension exists if that is the case.

This technique has been successfully applied to solve other extensions problems of this type (Hokari and Thomson, 2003; Dominguez and Thomson, 2006; Thomson, 2000, 2008a, 2008b, 2011).

2. Preliminaries

There is an infinite population of "potential" claimants indexed by $\mathbb{N}$, the natural numbers. Alternatively, we could assume this population to be finite and of cardinality at least 3. (If there are at most two agents, our variable-population axiom has no bite.) Let $\mathcal{N}$ be the class of finite subsets of $\mathbb{N}$. A **claims problem with agent set $N \in \mathcal{N}$** is a pair $(c, E) \in \mathbb{R}^N_+ \times \mathbb{R}_+$ where $c \equiv (c_i)_{i\in N}$ and $\sum_N c_i \geqslant E$. Each agent $i \in N$ has a **claim** $c_i \in \mathbb{R}_+$ over the **endowment** $E \in \mathbb{R}_+$; the endowment is insufficient to honor all of the claims. Let $\mathcal{C}^N$ denote the domain of all such problems. An **awards vector for (c, E)** is a vector $x \in \mathbb{R}^N_+$ satisfying the **claims boundedness** inequalities $x \leqq c$ and the **efficiency** equality $\sum_N x_i = E$.[4] Let $X(c, E)$ be the set of awards vectors for (c, E). A **rule** is a mapping defined over $\bigcup_{N\in\mathcal{N}} \mathcal{C}^N$ that associates with each problem an awards vector for it. Let S be our generic notation for rules.

For the two-(or three-)claimant case, a rule S is conveniently described in a two-(or three-)dimensional Euclidean space by fixing the claims vector c and determining the path followed by $S(c, E)$ as E increases from 0 to $\sum_N c_i$. We refer to it as the **path of awards of S for c**, and we denote it by $p^S(c)$. Our proofs mainly consist in uncovering geometric relations between paths of awards, in particular as the claimant set changes.

The notation (c'_i, c_{-i}) designates the vector obtained from c by replacing its i-th component by c'_i. Given $x^1, \ldots, x^k \in \mathbb{R}^N$ and $\ell \in \{1, \ldots, k-1\}$, $\mathbf{seg}[x^\ell, x^{\ell+1}]$ is the segment $\{y \in \mathbb{R}^N$: there is $\alpha \in [0, 1]$ such that $y = \alpha x^\ell + (1-\alpha)x^{\ell+1}\}$ and $\mathbf{bro.seg}[x^1, \ldots, x^k]$ is the broken segment $\text{seg}[x^1, x^2] \cup \cdots \cup \text{seg}[x^{k-1}, x^k]$. Given $A \subset \mathbb{R}^N$, $\mathbf{int}\{A\}$ is the interior of A relative to $\mathbb{R}^N_+$.

Next, we introduce several important rules, first for a fixed $N \in \mathcal{N}$. The first rule chooses awards proportional to claims. The second rule assigns amounts that are as equal as possible subject to no claimant receiving more than his claim. The third rule sets the losses experienced by all claimants as equal as possible subject to no claimant receiving a negative amount.[5] Here are the formulas, in which $N \in \mathcal{N}$, $(c, E) \in \mathcal{C}^N$, and $i \in N$ are arbitrary, and $\lambda \in \mathbb{R}_+$ is chosen so as to achieve efficiency. For the **proportional rule**, $P_i(c, E) \equiv \lambda c_i$. For the **constrained equal awards rule**, $CEA_i(c, E) \equiv \min\{c_i, \lambda\}$. For the **constrained equal losses rule**, $CEL_i(c, E) \equiv \max\{c_i - \lambda, 0\}$. To extend the rules to $\bigcup_{N\in\mathcal{N}} \mathcal{C}^N$, we add to the formulas a universal quantification over $N \in \mathcal{N}$. We will also need weighted versions of the last two rules.[6] Again, fix $N \in \mathcal{N}$. Let Δ^N

[3] In a variable-population model, each given agent may or may not be present.

[4] Conventions for vector inequalities: given $x, y \in \mathbb{R}^N$, $x \geqq y$ means $x_i \geqslant y_i$ for each $i \in N$; $x \geqslant y$ means $x \geqq y$ but $x \neq y$; $x > y$ means $x_i > y_i$ for each $i \in N$.

[5] O'Neill (1982), Aumann and Maschler (1985), Young (1987), and Dagan (1996) give references to ancient literature in which these rules are mentioned.

[6] One can certainly defined weighted versions of the proportional rule, but these rules violate our fixed-population axioms, even in the two-claimant case.