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The Dynamic Vickrey Auction

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ABSTRACT

We study the efficient allocation of a single object over a finite time horizon. Buyers arrive randomly over time, are long-lived, and have independent private values. The valuation of a buyer may depend on the time of the allocation in an arbitrary way. We construct an incentive compatible mechanism in which (A) there is a single financial transaction (with the buyer), (B) ex-post participation constraints are fulfilled, (C) there is no positive transfer to any agent and (D) payments are determined online. We exploit that under the efficient allocation rule, there is a unique potential winning period for each buyer. This reduces the multidimensional type to one dimension and the payment of the winner can be defined as the lowest valuation for the potential winning period, with which the buyer would have won the object. In a static model, this payment rule coincides with the payment rule of the Vickrey Auction.

1. Introduction

In many situations, the allocation of an object is a dynamic problem. Buyers arrive over time and have preferences regarding the time of the allocation. In addition to the winner determination, an allocation mechanism also has to determine the time of the allocation. Think for example of a government that wants to privatize an asset or a state-owned company—"the object"—in order to reduce its budget deficit, and wants to achieve an efficient allocation of the object. Potential buyers arrive over time and have preferences as to when they would like to acquire the object. For example, there may be investors whose business plan requires to start using the object immediately, while others are more patient. Others again may need some time before they can buy the asset and start using it, for example because complementary investments have not been made yet, or because they require some time to secure funding of the acquisition.

In this paper, a mechanism is constructed that implements the efficient allocation rule in such a dynamic environment and satisfies the following properties:

(A) There is a *single monetary transaction* which takes place between the buyer of the object and the mechanism.
(B) *Ex-post participation constraints* are fulfilled for all agents.
(C) There is *no positive transfer payment* to any participant in any period.
(D) All *transactions are made online*—that is, all information that is needed to determine the payment must be available at the time of the allocation.

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These properties are desirable from a practical point of view. Financial transactions with other agents than the winning bidder entail higher transaction costs. A mechanism that satisfies Property A therefore reduces transaction costs to a minimum. Properties B and C ensure that the mechanism can be implemented even if it is difficult or costly to prevent participants from reneging on their bids. If voluntary participation constraints are violated ex-post, the outcome, in particular payments required from the participants may be harder to enforce. For example, if loosing bidders have to make positive payments to the mechanism, they will try to withdraw their participation even if this entails a small cost. Similarly, a mechanism that regularly makes positive transfer payments will attract bidders who are not interested in the object and plan to withdraw their participation unless the outcome is that they receive a positive payment. A mechanism that satisfies Properties A to C therefore minimizes transaction costs or costs associated with the enforcement of the outcome of the mechanism. The last property (D) is indispensable in a dynamic allocation problem. If payments cannot be determined online, this means that additional information has to be collected from future buyers after the winner has already been determined. Incentives for reporting such information cannot be provided, however, because the allocation decision has already been made. Furthermore, online payments allow to match payments with delivery, which makes it easier to enforce payments.[2]

This paper analyzes the dynamic allocation of a single object over a finite time horizon. The model generalizes the standard independent private values framework. Potential buyers arrive randomly over time, they are long-lived, and their valuations for the object may depend on the time of the allocation in an arbitrary way. If the valuations did not depend on the timing of the allocation, the seller could wait for the last period and run a standard auction such as the Vickrey Auction (or the English Auction). In this auction format, the highest bidder wins and pays an amount equal to the second highest bid. If all bidders use their dominant strategies and place a bid equal to their valuation, an efficient allocation is achieved and moreover, Properties A to D are fulfilled.

In a dynamic framework, the existence of efficient mechanisms that satisfy these properties is not obvious. The static Vickrey–Clarke–Groves (Vickrey, 1961; Clarke, 1971; Groves, 1973) mechanism has been extended to various, quite general, dynamic settings (Athey and Segal, 2007; Bergemann and Välimäki, 2010; Cavallo et al., 2010; Parkes and Singh, 2003). The underlying principle of these dynamic versions of the VCG mechanism is that an agent who reports information that influences current or future allocation decisions, is required to pay an amount equal (in expectation) to the externality that her report imposes on all other agents. This implies that transactions are not limited to eventual buyers of the object. For example, a buyer A who reports that she is willing to pay a high price tomorrow, may induce the mechanism to store the object instead of selling it today to some impatient buyer B. Buyer A, however, will not get the object tomorrow if a third buyer C with a higher willingness to pay arrives. This is an example of a situation where, in contrast to a static allocation problem, a buyer's report influences the allocation decision even though she does not win the object. Without A's report, the object would not be allocated to C because the option value of storing the object would be lower than B's valuation for an allocation today. In the dynamic variants of the VCG mechanism proposed in the literature, such *pivotal* but non-winning buyers have to be compensated or charged for the externality, respectively, in order to ensure incentive compatibility. Parkes and Singh (2003) and Bergemann and Välimäki (2010) construct mechanisms in which payments are made online, but require transactions with multiple agents. The mechanisms also transfer positive amounts to participants, or violate ex-post participation constraints, respectively (see Section 4.4). For the independent private values framework, little is known about the existence of efficient mechanisms that satisfy all Properties A to D.[3]

In the mechanism proposed in this paper, only the winning bidder has to make a payment, which is equal to the lowest valuation with which she could have won the object. This valuation is called the *critical valuation* of the winner. In the static Vickrey Auction, the critical valuation is equal to the second highest valuation and coincides with the externality that the winner imposes on the other bidders.[4] In the dynamic framework this is not the case. The critical valuation is the maximum of (a) the second highest valuation for the period in which the bidder wins, (b) the option value of storing the object for future allocations, and (c) the lowest valuation for the winning period for which it is efficient to store the object instead of

[2] For a discussion of the advantages of Property D see also Cole et al. (2008) who distinguish *prompt mechanisms* that determine payments at the time of the allocation, and *tardy mechanisms* that defer payments to later times (see also Babaioff et al., 2010).

[3] Gershkov and Moldovanu (2009) and Gershkov et al. (2013) show that in dynamic environments where valuations or arrivals are correlated over time, the efficient allocation rule cannot always be implemented by a mechanism that satisfies all Properties A to D. Correlated types or arrivals lead to an informational externality of a buyer's report on the seller's option value of storing the object for future allocations. In general, this informational externality can only be reflected in the payments made by the buyer if the designer can condition payments on the realized types and arrivals of future buyers, which are drawn from the correct distribution, but such a scheme violates Property D (see also Mezzetti, 2004; Athey and Segal, 2007). In the case of correlated valuations, Gershkov and Moldovanu (2009) demonstrate that online payments are insufficient to implement the efficient allocation rule if the informational externalities are too strong. In the case of correlated arrivals Gershkov et al. (2013) show that Property D can be fulfilled at the expense of Property C.

[4] In models with one-dimensional private information, such as the standard static auction model, critical valuation payment schemes arise naturally when a deterministic allocation rule is implemented in dominant strategies. The Vickrey Auction as an implementation of the efficient allocation rule is the classic example. The emergence of such payment schemes can also be observed in more general settings, in particular in dynamic models where additional dimensions of private information such as an arrival time or an exit time are added (see Hajiaghayi et al., 2005). Note, however, that the arrival time and exit time are special because (a) they enter the utility function only as a constraint and (b) misreports are restricted to one direction. In the present model, agents have fully multidimensional valuations so that critical value payment schemes are not straightforwardly defined. Second, the present paper uses periodic ex-post equilibrium rather than dominant strategies as a solution concept so that other payment schemes could be used even in one-dimensional models or the restricted generalizations discussed before.