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Finite supermodular design with interdependent valuations ☆

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ABSTRACT

This paper studies supermodular mechanism design in environments with arbitrary (finite) type spaces and interdependent valuations. In these environments, the designer may have to use Bayesian equilibrium as a solution concept, because ex-post implementation may not be possible. We propose direct (Bayesian) mechanisms that are robust to certain forms of bounded rationality while controlling for equilibrium multiplicity. In quasi-linear environments with informational and allocative externalities, we show that any Bayesian mechanism that implements a social choice function can be converted into a supermodular mechanism that also implements the original decision rule. The proposed supermodular mechanism can be chosen in a way that minimizes the size of the equilibrium set, and we provide two sets of sufficient conditions to this effect. This is followed by conditions for supermodular implementation in unique equilibrium.

1. Introduction

This paper studies supermodular mechanism design in environments with interdependent valuations and arbitrary (in particular, multidimensional) finite type spaces. This approach was introduced by Mathevet (2010) in differentiable quasi-linear environments with private values and one-dimensional types.[1] The main motivation is to design direct mechanisms that are robust to certain forms of bounded rationality while controlling for equilibrium multiplicity. It is important to extend supermodular mechanism design to environments with informational and allocative externalities and multidimensional types for at least two reasons. First, these environments capture many realistic situations. Second, it is often impossible to use dominant strategy or ex-post implementation in these settings (see Jehiel et al., 2006, and Section 2), and thus the designer may resort to Bayesian equilibrium as a solution concept. It becomes useful to have a simple method for improving the behavioral robustness of Bayesian mechanisms.

In this paper, we are concerned with the design of supermodular mechanisms whose equilibrium set is of minimal size. We call this minimal supermodular implementation. Supermodular mechanism design aims to induce the right incentives so that agents play a supermodular game. Supermodular games are games where players have monotone best responses, i.e. each player wants to play a "larger" strategy if others do so as well. On the theoretical front, the reasons for using supermodular mechanisms stem from Milgrom and Roberts (1990, 1991) and Vives (1990): supermodular games have extremal equilibria, a smallest and a largest one, that enclose all the iteratively undominated strategies and all the limit points of all adaptive and sophisticated learning dynamics. Therefore, supermodular games are robust to a wide range of behaviors,

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[1] Chen (2002) was the first to propose a supermodular mechanism (to implement the Lindahl correspondence).

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including boundedly rational behaviors. In particular, if the designer had the opportunity to use her mechanism repeatedly, then adaptive learners (Milgrom and Roberts, 1990) would end up within the interval prediction, which is the interval between the extremal equilibria. Therefore, the objective of minimizing the size of the interval prediction has several virtues. It minimizes the multiple equilibrium problem, since all equilibria are contained in it.[2] It also guarantees a more accurate convergence of the learning dynamics. Ideally, this interval reduces to a single point in certain situations (see Section 2), thereby solving the multiplicity issue and ensuring convergence of all dynamics.

Supermodular mechanisms have other attractive theoretical properties. Not only are their mixed strategy equilibria unstable (Echenique and Edlin, 2004), which justifies ruling them out of the analysis, but many pure equilibria are stable, such as the extremal equilibria (Echenique, 2000). Thus, a perturbation should not destabilize permanently a socially desirable alternative implemented via a supermodular mechanism (provided the underlying equilibrium was stable).

The robustness properties of supermodular mechanisms have been corroborated by several experiments. Chen and Gazzale (2004) run experiments on a game for which they control the amount of supermodularity. They show how convergence in that game is significantly better when it is supermodular. Healy (2006) tests five public goods mechanisms and he observes that subjects learn to play the equilibrium only in those mechanisms that induce a supermodular game. Other experiments (e.g. Chen and Plott, 1996, and Chen and Tang, 1998) provide results emphasizing the importance of dynamic convergence in the context of implementation. Most of these experiments demonstrate that convergence to an equilibrium is not a trivial issue.

In this paper, we generalize supermodular mechanism design to environments with allocative externalities, interdependent valuations (i.e. informational externalities) and arbitrary (finite) type spaces. There are two important reasons for doing so.

Firstly, it allows us to cover mechanism design problems of interest. The importance of allocative externalities is well documented in the literature. Jehiel and Moldovanu (1996) use patent licensing in an oligopolistic market as an example. Informational externalities are also a realistic assumption, proved to be interestingly challenging by many papers (Cremer and McLean, 1985; Maskin, 1992; Dasgupta and Maskin, 2000; Perry and Reny, 2002; Chung and Ely, 2002; Bergemann and Morris, 2003, etc.). Finally, it is often natural to interpret information as a multidimensional type in many design problems. Consider, for example, oil companies bidding to obtain a drilling permit. Their private information is modeled as a multidimensional signal (e.g., expected amount of oil in the oil field, proximity to other reserves, etc.).

Secondly, it is difficult to use dominant strategy or ex-post implementation in those environments — with allocative externalities, interdependent valuations and multidimensional types — and thus behaviorally-robust Bayesian mechanisms become especially appealing. In quasi-linear environments with interdependent valuations and multidimensional types, many impossibility results limit the set of available solution concepts. The conclusions are rather pessimistic about dominant strategy equilibrium and ex-post equilibrium. Williams and Radner (1988) show that efficient dominant strategy implementation is generally not possible when agents have interdependent valuations. Jehiel et al. (2006) prove a strong impossibility result: when types are multidimensional and valuations are interdependent, only trivial decision rules can generically be implemented in ex-post equilibrium. If the designer wants to implement a meaningful social choice function, not even necessarily efficient, she may have to use Bayesian equilibrium as a solution concept (see Section 2). Even then, impossibility results exist. Jehiel and Moldovanu (2001) show that it is difficult to reconcile Bayesian incentive compatibility with some efficiency constraint. These negative results indicate that Bayesian equilibrium may often be a natural candidate as a solution concept. However, playing a Bayesian equilibrium requires more, in general, on the part of the agents. Agents have to be Bayesian rational, and strong knowledge assumptions about the information structure and the rationality must hold (Brandenburger and Dekel, 1989). For example, Bayesian equilibrium calls for correct predictions of opponents' play to determine one's own strategy. In this context, the ability to construct supermodular Bayesian mechanisms is attractive, because eventual play of some equilibrium can be achieved by unsophisticated agents who follow simple behavioral rules.

Our paper provides methods for converting *any* truthful Bayesian mechanism into a (truthful) supermodular mechanism whose equilibrium set is of minimal size. The idea is to create complementarities between agents' announcements by augmenting the original transfer scheme with a function. This function vanishes in expectation and therefore preserves incentive compatibility. Although there exist many ways in which a mechanism can be transformed into a supermodular mechanism, we derive the one that most adequately addresses the multiple equilibrium problem. To this purpose, we add just enough strategic complementarities to ensure that a supermodular game is induced, but not in any excess of that level.

We present two sets of results for minimal supermodular implementation. In both instances, "best" is used to designate the mechanism with the smallest interval prediction. The first result shows that if a social choice function is implementable, then its decision rule can be implemented by the best supermodular mechanism among all the supermodular mechanisms *whose transfers are in a certain class*. No additional condition is required. In particular, this result holds for all (implementable) decision rules and all valuation functions. The result also provides an explicit transfer scheme.

The second result characterizes the *overall* best supermodular mechanism among all possible supermodular mechanisms or transfers: if a social choice function is implementable, then its decision rule can be implemented by the (overall) best supermodular mechanism if and only if some (explicitly stated) finite system of linear equations admits a solution. This

[2] If the outcome function of the mechanism is continuous and if the interval prediction is tight, then all equilibrium outcomes are close, so that the output of the mechanism must be close to the socially desirable objective.