Games and Economic Behavior 82 (2013) 444–467

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Strategic information exchange ☆

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ARTICLE INFO

Article history:
Received 24 December 2010
Available online 8 August 2013

JEL classification:
C72
C73
D82

Keywords:
Repeated games
Incomplete information on both sides
Information externalities
Folk Theorem
Equilibrium

ABSTRACT

We analyze a toy class of two-player repeated games with two-sided incomplete information. In our model, two players are facing independent decision problems and each of them holds information that is potentially valuable to the other player. We study to what extent, and how, information can be exchanged at equilibrium. We show that, provided one's initial information is valuable to the other player, equilibria exist at which an arbitrary amount of information is exchanged at an arbitrary high rate. The construction relies on an indefinite, reciprocated, exchange.

0. Introduction

Discounted repeated games with incomplete information are not quite well-understood yet. In the zero-sum framework of Aumann and Maschler (1995, 1966), Mayberry (1967) exhibits an example in which the value depends in a complex way on the discount factor. Cripps and Thomas (2003) and Peski (2008) look at games with *one-sided* information, in which each of the two players knows his own payoff function, and one of the two is unsure of the payoff function of the other player. Cripps and Thomas (2003) prove that a Folk Theorem type of result holds in the limit where the prior belief converges to the case of complete information. Peski (2008) essentially shows that all equilibria are payoff-equivalent to equilibria that involve finitely many rounds of information revelation. Wiseman (2005) looks at situations of *common* uncertainty. Players share the same information on the underlying state of nature, and refine this information by observing actual choices and payoffs. Hörner and Lovo (2009), and Hörner et al. (2011) provide a characterization of the set of belief-free equilibrium payoffs.

Our main goal in this paper is to analyze a toy class of simple-looking, yet thought-provoking, games with two-sided incomplete information. Consider two agents facing independent repeated decision problems. The two agents are unrelated, except that each has private information that is valuable for the other. Players cannot communicate, except through actions, which is costly. Can information be exchanged at equilibrium?

As an illustration, consider the following repeated, two-player, discounted game. Two biased coins C_1 and C_2 are tossed independently, once, at the outset of the game. The parameter of each coin is equal, say, to $\frac{2}{3}$. Each player i has to repeatedly

☆ The research of Solan was supported in part by ISF grant #212/09 and by the Google Inter-University Center for Electronic Markets and Auctions. Rosenberg and Vieille thank the ANR Explora for financial support.
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0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2013.07.010

guess the outcome of the coin C_i. A correct guess yields a payoff of 1, while an incorrect guess yields 0. We assume that only past actions are observed along the play, so that there is no room for statistical learning/experimentation.[1] In addition, we assume away cheap talk.

To make the game non-trivial, assume that once coins are tossed, each player gets to learn *only* the outcome of the *other player's* coin. This private information has no 'direct' value, since C_1 and C_2 are independent. In particular, it is an equilibrium for both players to ignore their private information and to repeat throughout the action that matches the most likely outcome of the coins. No information is ever exchanged, and each player's expected payoff is equal to $\frac{2}{3}$. It is readily checked that this is the unique belief-free equilibrium payoff.

Since cheap talk is assumed away, private information can be disclosed to the other player only through one's own actions. That is, disclosing information requires that a player condition his play on the other coin's outcome, and thus, play both of his actions with some positive probability. But one of these actions will typically be myopically suboptimal, in that it will yield a lower expected stage payoff than the other. Therefore, revelation of information is costly, and the cost depends on the player's belief on the outcome of his own coin.[2]

To illustrate this, let us ask whether a player, say player i, might be willing to 'tell' the outcome of C_j to player j at stage 1? The answer is plainly negative: if player j expects to be told the outcome of C_j in stage 1, his unique best response is to play his myopically optimal action at stage 1, and from stage 2 on to play according to the information received from player i in stage 1. But such a strategy does not provide player i with any information on the outcome of C_i. Therefore, player i will refuse to reveal any information in stage 1.

More generally, no player is willing to disclose information, unless he expects to be reciprocated later with valuable information. That is, no player is willing to be the last one in disclosing information.[3] This suggests that equilibria improving upon $(\frac{2}{3}, \frac{2}{3})$ *must* involve an open-ended, gradual exchange of valuable information. In Section 3, we show how to construct such equilibria and to implement a whole range of equilibrium payoffs.

The above example is one of a two-player repeated game with incomplete information on both sides and pure informational externalities – a player's payoff depends on the other player's information, but not otherwise on the other player's action. In this paper we characterize the limit set of sequential equilibrium payoffs for such games. Formally, a player's payoff depends only on his type and on his own action. Both types are drawn independently by nature at the outset of the game; the players receive private signals on the types' realizations, and the types remain fixed throughout the game. Along the play, players repeatedly choose actions, which are publicly disclosed.

Our model is admittedly quite specific, in that it rules out direct strategic interaction. This assumption of pure informational externalities plays a dual role. On one hand, it simplifies the analysis of the model and allows us to study the exchange of information in isolation from other strategic considerations. On the other hand, such games are games in which we would *least* expect that exchange of information might take place. Our main purpose is to come up with new tools and ideas, and we hope that any positive result in this highly non-generic setup may potentially pave the way for the analysis of other, more economically relevant setups.

Our main result is the following. We prove that, provided that the information held by each player is valuable to the other player, the limit set of sequential discounted equilibrium payoffs when players become more and more patient coincides with the set of all feasible payoffs, that are equal to or larger than the initial, myopic optimal payoffs. In the simple example discussed above, this limit set is thus equal to the set $[\frac{2}{3}, 1] \times [\frac{2}{3}, 1]$. That is, not only information can be shared, but the rate of information exchange can be arbitrarily high relative to the discount rate. Our equilibria share the following features. Players start by reporting truthfully whatever information they received on their own state. This leads to a continuation game in which no player holds private information on his own state. As a result, each player is able to compute how costly it is for the other player to play his suboptimal action, and is therefore able to fine-tune his information disclosure policy, so as to provide the other player with appropriate incentives for disclosing information.

Players next exchange information in an open-ended manner. The analysis presents two main and mostly independent difficulties. One is to design open-ended equilibrium processes, according to which information is exchanged. In our construction, the bulk of information exchange takes place early in the game. Later information disclosure serves only as a means to compensate for previously incurred costs. The second difficulty consists in adjusting this continuation play so as to provide the incentives for truthfully reporting one's information on one's own state.

Starting with Crawford and Sobel (1982), the huge literature on strategic information transmission and on cheap-talk games addresses issues related to ours. The paper that is closest to our work is Aumann and Hart (2003). There, prior to playing a game once, two players, one of which is informed of the true game to be played, exchange messages during countably many periods. Aumann and Hart (2003) characterize the set of equilibrium payoffs. Following an example of Forges (1990), they show that allowing for an unbounded communication length may increase the set of equilibrium payoffs. Results of Aumann and Hart (2003) were extended by Amitai (1996) to cheap talk with two-sided incomplete information. In particular, Amitai shows that the set of equilibrium payoffs depends on the size of the message space. There are however

[1] This assumption is discussed at length in, e.g., Mertens (1987).

[2] The variant in which players are allowed to exchange messages at a fixed cost leads to an analysis similar to that of the present paper, as we let the cost of messages vanish.

[3] As is shown later, the fact that a player may be indifferent between both actions does not open up new possibilities.