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Anonymous social influence ☆

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ABSTRACT

We study a stochastic model of influence where agents have "yes" or "no" inclinations on some issue, and opinions may change due to mutual influence among the agents. Each agent independently aggregates the opinions of the other agents and possibly herself. We study influence processes modeled by ordered weighted averaging operators, which are anonymous: they only depend on how many agents share an opinion. For instance, this allows to study situations where the influence process is based on majorities, which are not covered by the classical approach of weighted averaging aggregation. We find a necessary and sufficient condition for convergence to consensus and characterize outcomes where the society ends up polarized. Our results can also be used to understand more general situations, where ordered weighted averages are only used to some extent. Furthermore, we apply our results to fuzzy linguistic quantifiers, i.e., expressions like "most" or "at least a few".

1. Introduction

In the present work we study an important and widespread phenomenon which affects many aspects of human life – the phenomenon of *influence*. Being undoubtedly present, e.g., in economic, social and political behaviors, influence frequently appears as a dynamic process. In particular, social influence plays a crucial role in the formation of opinions and the diffusion of information and thus, it is not surprising that numerous scientific works investigate different dynamic models of influence.[1]

Grabisch and Rusinowska (2010, 2011) investigate a one-step deterministic model of influence, where agents have "yes" or "no" inclinations (beliefs) on a certain issue and their opinions may change due to mutual influence among the agents. Grabisch and Rusinowska (in press) extend it to a dynamic stochastic model based on aggregation functions, which determine how the agents update their opinions depending on the current opinions in the society. Each agent independently aggregates the opinions of the other agents and possibly herself. This aggregation determines the probability that "yes" is her updated opinion after one step of influence (and otherwise it is "no"). The other agents only observe this updated

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[1] For an overview of the vast literature on influence we refer, e.g., to Jackson (2008).

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opinion. Since any aggregation function is allowed when updating the opinions, the framework covers numerous existing models of opinion formation. The only restrictions come from the definition of an aggregation function: unanimity of opinions persists (boundary conditions) and influence is positive (nondecreasingness). Grabisch and Rusinowska (in press) provide a general analysis of convergence in the aggregation model and find all terminal classes, which are sets of states the process will not leave once they have been reached. Such a class could only consist of one single state, e.g., the states where we have unanimity of opinions ("yes"- and "no"-consensus) or a state where the society is polarized, i.e., some group of agents finally says "yes" and the rest says "no".

Due to the generality of the model of influence based on arbitrary aggregation functions introduced in Grabisch and Rusinowska (in press), it would be difficult to obtain a deeper insight into some particular phenomena of influence by using this model. This is why the analysis of particular classes of aggregation functions and the exhaustive study of their properties are necessary for explaining many social and economic interactions. One of them concerns *anonymous social influence* which is particularly present in real-life situations. Internet, accompanying us in everyday life, intensifies enormously anonymous influence: when we need to decide which washing machine to buy, which hotel to reserve for our eagerly awaited holiday, we will certainly follow all anonymous customers and tourists that have expressed their positive opinion on the object of our interest. In the present paper we examine a particular way of aggregating the opinions and investigate influence processes modeled by *ordered weighted averaging operators* (*ordered weighted averages*), commonly called *OWA operators* and introduced in Yager (1988), because they appear to be a very appropriate tool for modeling and analyzing anonymous social influence. Roughly speaking, OWA operators are similar to the ordinary weighted averages (weighted arithmetic means), with the essential difference that weights are not attached to agents, but to the *ranks* of the agents in the input vector. As a consequence, OWA operators are in general nonlinear, and include as particular cases the median, the minimum and the maximum, as well as the (unweighted) arithmetic mean.

We show that OWA operators are the only aggregation functions that are *anonymous* in the sense that the aggregation does only depend on how many agents hold an opinion instead of which agents do so. Accordingly, we call a model *anonymous* if the transitions between states of the process do only depend on how many agents share an opinion. We show that the concept is consistent: if all agents use anonymous aggregation functions, then the model is anonymous. However, as we show by example, a model can be anonymous although agents do not use anonymous functions. In particular, anonymous models allow to study situations where the influence process is based on *majorities*, which means that agents say "yes" if some kind of majority holds this opinion.[2] These situations are not covered by the classical (commonly used) approach of weighted averaging aggregation.

In the main part, we consider models based on OWA operators. We discuss the different types of terminal classes and characterize terminal states, i.e., singleton terminal classes. The condition is simple: the OWA operators must be such that all opinions persist after mutual influence. In our main result, we find a necessary and sufficient condition for convergence to consensus. The condition says that there must be a certain number of agents such that if at least this number of agents says "yes", it is possible that after mutual influence more agents say "yes" and if less than that number of agents says "yes", it is possible that after mutual influence more agents say "no". In other words, we have a cascade that leads either to the "yes"- or "no"-consensus. Additionally, we also present an alternative characterization based on *influential coalitions*. We call a coalition influential on an agent if the latter follows (adopts) the opinion of this coalition – given all other agents hold the opposite opinion – with some probability.[3] Furthermore, we generalize the model based on OWA operators and allow agents to use a (convex) combination of OWA operators and general aggregation functions (*OWA-decomposable* aggregation functions). In particular, this allows us to combine OWA operators and ordinary weighted averaging operators. As a special case of this, we study models of mass psychology (also called herding behavior) in an example. We find that this model is equivalent to a convex combination of the majority influence model and a completely self-centered agent. We also study an example on *important agents* where agents trust some agents directly that are important for them and otherwise follow a majority model. Furthermore, we show that the sufficiency part of our main result still holds.[4]

As an application of our model we study *fuzzy linguistic quantifiers*, which were introduced in Zadeh (1983) and are also called *soft quantifiers*. Typical examples of such quantifiers are expressions like "almost all", "most", "many" or "at least a few"; see Yager and Kacprzyk (1997). For instance, an agent could say "yes" if "most of the agents say 'yes' ".[5] Yager (1988) has shown that for each quantifier we can find a unique corresponding OWA operator.[6] We find that if the agents use quantifiers that are similar in some sense, then they reach a consensus. Moreover, this result holds even if some agents deviate to quantifiers that are not similar in that sense. Loosely speaking, quantifiers are similar if their literal meanings are "close", e.g., "most" and "almost all". We also give examples to provide some intuition.

We terminate this section with a very brief overview of the related literature. One of the main differences between our work and the existing models on opinion formation lies in the way agents are assumed to aggregate the opinions. Except,

[2] Examples are simple majorities as well as unanimity of opinions, among others.

[3] Note that although Grabisch and Rusinowska (in press) have already studied conditions for convergence to consensus and other terminal classes in the general model, our results are inherently different due to our restriction to anonymous aggregation functions.

[4] When applying the condition to the OWA operators in the convex combinations.

[5] Note that the formalization of such quantifiers is clearly to some extent ambiguous.

[6] With the only restriction that, due to our model, the quantifier needs to represent positive influence.