A dynamic school choice model

Juan Sebastián Pereyra

El Colegio de México, Centro de Estudios Económicos, Camino al Ajusco no. 20, Pedregal de Santa Teresa, 10740 México DF, Mexico

A B S T R A C T

This paper inspires from a real-life assignment problem faced by the Mexican Ministry of Public Education. We introduce a dynamic school choice problem that consists in assigning positions to overlapping generations of teachers. From one period to another, teachers can either retain their current positions or choose a preferred one. In this framework, a solution concept that conciliates the fairness criteria with the individual rationality condition is introduced. It is then proved that a solution always exists and that it can be reached by a modified version of the deferred acceptance algorithm of Gale and Shapley. We also show that the mechanism is dynamically strategy-proof, and respects improvements whenever the set of orders is lexicographic by tenure.

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1. Introduction

Since David Gale and Lloyd Shapley published their famous paper "College admissions and the stability of marriage" (Gale and Shapley, 1962), many authors have studied assignment problems in different contexts. Therefore, there is an extensive literature on allocation problems, which primarily considers static models. In contrast, there are many real-life applications where the assignment is made in a dynamic context. Some examples are on-campus housing for college students, in which freshmen apply to move in and graduating seniors leave (Kurino, 2011), kidney exchange of patients, in which each agent arrives with an object to trade (Ünver, 2010), and firms with workers whose entry and exit lead to a reassignment of fixed resources (Bloch and Cantala, 2013). In this paper we study a dynamic version of the well-known school choice model. Specifically, our model assigns school positions to overlapping generations of teachers. In each period, the central authority must assign positions to teachers, taking into account each school's priority ranking and the previous matching. From one period to another, teachers can either retain their current positions or choose a preferred one. Hence, the central authority faces a dynamic allocation problem.

The original motivation for this paper is an assignment problem faced by the Mexican Ministry of Public Education. In May 2008 the Mexican Federal Government, through the Ministry of Public Education, signed an agreement with the National Education Workers Union called "The Alliance for the Quality of Education".[1] Part of the agreement was the creation of the National Contest for the Allocation of Teaching Positions, a mechanism to assign teachers to teaching positions. As a consequence of this agreement, teachers looking for a position in the public education system are required to sit an exam. According to each teacher's grade, the central authority ranks teachers and then assigns each a teaching position. Specifically,

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under the mechanism used by the central authority, all open positions (that is, positions that are not already assigned) are offered to the first teacher in the ranking. Once this first ranked teacher chooses a school, remaining open positions are offered to the second teacher, and so on. Moreover, any teacher that had been previously assigned a position may choose to retain it over the new positions that are offered, but if she chooses a new one, her previous position becomes "open". In Appendix A we present an example with an application of this mechanism. It is worth noting that the algorithm is not a sort of "You-request-my-house-I-get-your-turn" (Abdulkadiroğlu and Sönmez, 1999) since a position that was assigned to a teacher in a previous period, cannot be asked by another teacher until the teacher who is assigned to it, moves to another position. Thus, the central authority applies a variant of the serial dictatorship mechanism, which takes into account that some teachers are initially assigned a position. In 2012, 134,704 teachers participated in the exam in order to obtain a position.

Cantala (2008) shows that the mechanism has some major flaws (see Appendix A for an illustrative example). In particular, a teacher can profit in a period after she enters the market by misrepresenting her preferences. This implies that the mechanism is not dynamically strategy-proof: it can be manipulated by teachers. Another flaw is that the mechanism does not respect improvements made by teachers (Balinski and Sönmez, 1999), that is, a teacher may increase her order in one school's priority ranking, but be assigned to a worse position. Finally, Cantala (2008) shows that the mechanism is not efficient. In this paper, we study the described problem within a more general framework in order to cast some light on the resource allocation problem faced by the Mexican Ministry of Public Education.

A central concept in matching theory is stability: a matching is stable if there does not exist any unmatched teacher–school pair (i, s) such that i prefers s to the school that she is assigned to and there exists a teacher assigned to s who has a lower priority at s than i. In school choice models, this concept is usually referred to as the elimination of justified envy (Abdulkadiroğlu and Sönmez, 2003) and embodies a notion of fairness. In addition to elimination of justified envy, since we cannot assign a teacher to a less preferred school than the one where she is teaching, we have to address the individual rationality condition. We present a new solution concept to accommodate these concepts.

In order to define our solution concept, we consider the claims that could exist in a matching. A teacher has a claim over a school if there exists a school that she prefers over her assignment, and she has higher priority for it than one of the assigned teachers. Note that a matching eliminates the justified envy if and only if there is no claim in the matching. Moreover, we consider two kinds of claims. If the teacher in the preferred school was not assigned to it in the previous period, we say that it is a justified claim. On the contrary, if the teacher was assigned to the school in the previous period, the claim is considered inappropriate. Observe that the last type of claim is inappropriate due to the individual rationality restriction. Finally, our solution concept is as follows. We say that a matching minimizes inappropriate claims if:

– it is individually rational, non-wasteful (whenever a teacher prefers a school to her own assignment, that school already has all its positions filled), and does not have justified claims; and
– if there are inappropriate claims, the following must hold: there is no other matching that satisfies the three previous properties and one inappropriate claim is *solved* without creating a new one.

It is worth noting that Mexican Ministry of Public Education did not propose an explicit fairness concept and, also, that the mechanism which is used by this central authority does not satisfy the last definition.

In this context, we show that within the set of matchings that minimize inappropriate claims, there is a unique matching Pareto superior to all other matchings. In order to find it, a modified version of the deferred acceptance algorithm of Gale and Shapley is introduced. Before applying the algorithm, we modify each school's priority ranking by moving teachers who had been assigned to the school in the previous period to the top of the school's priority ranking.[2] With these new orders we define the related market in which the deferred acceptance algorithm is applied.

A new dynamic version of strategy-proofness is introduced. The classic concept in static matching problems only makes reference to the benefit in one period. Our notion of strategy-proofness is dynamic in the sense that it involves not only the period when the teacher enters the market but also all the later periods while she is in the market. In our framework, teachers reveal their preferences in the period in which they enter the market. In the following periods, they cannot modify the announced preferences. We prove that if each school's priority ranking is lexicographic by tenure, that is, if teachers who were present in the previous period have priority over new teachers, then the proposed mechanism is dynamically strategy-proof. Finally, it is shown under the same condition that the mechanism also respects improvements made by teachers. Our concept of respecting improvements involves not only the period when the teacher improves her position in the ranking (like the classic notion), but also every following period.

Our model assumes that teachers' preferences are time invariant, that is, teachers reveal their preferences entering the market and do not change from one period to the other. Although the assumption of time-invariant preferences is strong in many real-life applications, we think that it fits the market studied in the paper. The assumption reflects two teachers' behavioral patterns present in this market. In the first place, we know that most teachers that work in rural areas want to settle in a big city. The reason for this behavior is that residing in a big city implies better standards of living and more

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