



The e-mail game phenomenon [☆]

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ABSTRACT

The e-mail game in Rubinstein (1989) shows that types with arbitrarily close higher-order beliefs may differ substantially in strategic behaviors. We define a notion called strategic discontinuity in arbitrary incomplete-information scenarios to generalize this e-mail game phenomenon. We show that almost all types involved in economic analysis – types in finite or common-prior models – display strategic discontinuity in simple games.

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1. Introduction

From the very beginning of the theory of games with incomplete information, it has been understood that in order to analyze the strategic behavior in the face of uncertainty, the model has to specify the players' beliefs, their beliefs about the beliefs of their opponents, beliefs about their opponents' beliefs about their own beliefs, and so on. These higher-order beliefs matter for the behavior, which is best demonstrated in the e-mail game due to Rubinstein (1989). Specifically, Rubinstein constructs a simple 2×2 coordination game with two strict Nash equilibria, as well as an incomplete-information scenario in which the players mutually know the payoffs only up to a finite level k .¹ When k is large, the mutual-knowledge scenario seems to “approximate” the common-knowledge (i.e., complete-information) scenario, yet only one of the two equilibria is (ϵ -)rationalizable in the mutual-knowledge scenario.²

The e-mail game provides “a useful, if extreme, illustration of the logic by which higher-order beliefs and knowledge might influence outcomes in strategic settings (Morris, 2002, p. 434)”. Furthermore, it has by now inspired a large literature

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¹ Throughout the paper, we consider two-player games. An $m \times n$ game is a game in which player 1 has m actions and player 2 has n actions.

² The literature on global games initiated by Carlsson and Van Damme (1993) exploits a similar idea to argue that introducing a vanishingly small noisy signal (similar to the approximation of types which we study in this paper) in a game with multiple equilibria selects a unique outcome.

attempting to explore its different aspects.³ Since it is difficult in practice, if possible, to sort out the precise specification of players' higher-order beliefs, the e-mail game highlights a fundamental caveat on the way we analyze and predict strategic behaviors.

However, many games involved in economic analysis are neither 2×2 games nor games with complete information. If the e-mail game phenomenon occurs only in such a specific environment, whatsoever it suggests would be less of a concern. For a modeler, the issue is to what extent should she be worried about the phenomenon. That is, it would be useful to learn which sort of higher-order beliefs will lead to the e-mail game phenomenon; in which kind of games the phenomenon occurs; and above all, whether the e-mail game represents a general situation or merely a special case. We attempt to answer these questions in the paper.

We use (Harsanyi) types to formulate incomplete-information scenarios. Each type of a player identifies the player's belief about the payoff-relevant states (i.e., the first-order belief), the player's belief about the other player's beliefs about the payoff-relevant states (i.e., the second-order belief), and so on. We adopt the solution concept of interim correlated rationalizability (ICR) proposed in Dekel et al. (2006, 2007) to model strategic behaviors.⁴ We define a notion called *strategic discontinuity* to generalize the e-mail game phenomenon. We say that type t displays strategic discontinuity in game G if there exist $\varepsilon > 0$ and a sequence of types $\{t_m\}_{m=1}^{\infty}$ such that (i) $\{t_m\}_{m=1}^{\infty}$ approximates t in the sense that t_m and t share the same m th-order belief for every m ; (ii) some action in G is rationalizable for t but not ε -rationalizable for any t_m .

We say a type is n -critical if it displays strategic discontinuity in an $n \times n$ game.⁵ For instance, the common-knowledge type in the e-mail game is 2-critical. Furthermore, a type is *critical* if it is n -critical for some finite n . Ely and Pęski (2011) fully characterize the set of critical types in terms of the notion of *common- p beliefs* due to Monderer and Samet (1989) and show that almost all "economic types" are critical. However, the games they construct to demonstrate the strategic discontinuity are complicated and require a large number of actions. This is in sharp contrast to the simple 2×2 game employed by Rubinstein (1989) and the prevalence of simple games in economic applications, which motivates our study of the strategic discontinuity in simple games, i.e., n -critical types with a small n .

We start by analyzing finite types, i.e., types in a finite type space. Such types appear frequently in both applied and theoretical economic literature. Our first result (Theorem 1) shows that every finite type is 3-critical. More precisely, every finite type displays strategic discontinuity in some game in a one-dimensional class of 2×3 coordination games. We then extend the argument to the larger family of types among which players hold common- p belief about a proper closed subset of first-order beliefs. This family includes all finite types and almost all types admitting common priors. We prove that every type in this family displays strategic discontinuity in some game in a two-dimensional class of 2×4 games and is thus 4-critical (Proposition 1). Consequently, our second main result (Theorem 2) shows that every common prior assigns probability one to 4-critical types. Finally, for every positive integer n , we prove that critical types which are n -regular exist (Theorem 3). That is, even though almost all of the commonly-seen types display strategic discontinuity in simple games, it is necessary for Ely and Pęski (2011) to consider complicated games to demonstrate the strategic discontinuity for all critical types.

The simple games that we use to demonstrate strategic discontinuity resemble the e-mail game. Our result thus implies that the e-mail game phenomenon arises around all finite types and types which admit common priors. As a result, even though these "economic types" are generally more complicated to describe and thus more permissive for modeling strategic situation, they are almost as fragile as the common-knowledge type in Rubinstein (1989). More precisely, in all of the applied economic models which assume a finite and/or common-prior type space, even if the modeler can confine herself to simple (2×3 or 2×4) games, there is no guarantee that the model conclusion is robust to misspecification of higher-order beliefs.

The rest of the paper proceeds as follows. Section 2 provides formal notations and definitions. Section 3 presents the main results. Section 4 concludes. Technical proofs are relegated to Appendix A.

2. Preliminaries

Throughout the paper, we fix a two-player set I and a finite set of payoff-relevant states (i.e., the space of fundamental uncertainty) Θ which contains at least two elements.⁶ Given a player $i \in I$, we write $-i$ to denote the other player in I . For every metric space Y , let $\Delta(Y)$ be the space of all probability measures on the Borel σ -algebra of Y . We endow $\Delta(Y)$ with the weak*-topology. Let $\text{supp } \mu$ be the support of a probability measure μ , i.e., the intersection of all closed sets with measure one under μ . For a measure μ on the product space $Y_1 \times Y_2$, we use $\text{marg}_{Y_1} \mu$ to denote the marginal distribution

³ For instance, there have been numerous works in experimental and behavioral economics using the e-mail game phenomenon to motivate and study new models of reasoning in games (see, e.g., the recent paper by Strzalecki, 2009 and references therein).

⁴ All of our results remain true if we adopt instead the solution concept of interim independent rationalizability studied in Morris and Skidas (2000), Battigalli and Siniscalchi (2003), and Ely and Pęski (2006).

⁵ In this paper, we follow Dekel et al. (2006), Ely and Pęski (2011), Chen et al. (2010, 2012) to consider a class of games associated with a fixed set of payoff-relevant states Θ . More precisely, for different n -critical types, we demonstrate the strategic discontinuity by suitably varying the payoffs defined on the $n \times n \times |\Theta|$ profiles of actions and Θ . An alternative way to generalize the e-mail game is taken by Weinstein and Yildiz (2007), which studies the strategic discontinuity in a fixed game satisfying a *richness* condition. In Chen and Xiong (2011), we point out and study a difference between the e-mail game phenomenon and the notion studied in Weinstein and Yildiz (2007).

⁶ All of our results can be easily extended to n -player games with any finite $n \geq 3$.

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