Games and Economic Behavior 80 (2013) 229–242

Contents lists available at SciVerse ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Optimal mechanism design for the private supply of a public good

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ARTICLE INFO

Article history:
Received 28 June 2012
Available online 24 April 2013

JEL classification:
C61
C72
D82
H41

Keywords:
Public good provision
Mechanism design
Profit maximization

ABSTRACT

We study the problem of finding the profit-maximizing mechanism for a monopolistic provider of a single, non-excludable public good. Our model covers the most general setting, namely, we allow for correlation in the signal distribution as well as for informational externalities in the valuations. We show that the optimal deterministic, ex-post incentive compatible, ex-post individual rational mechanism can be computed in polynomial time by reducing the problem to finding a maximal weight closure in a directed graph. Node weights in the graph correspond to conditional virtual values, while the network structure is arising from the monotonicity constraints. We discuss what can be achieved if we relax our core assumptions one by one, i.e., if we go for randomized, interim individual rational or Bayes–Nash implementable mechanisms. Finally, we demonstrate that our techniques can be adapted for the excludable public good problem as well.

1. Introduction

We study the problem of finding a mechanism that maximizes the expected profit of a monopolistic provider of a single, non-excludable public good. The problem was first solved by Güth and Hellwig (1986) for the case of Bayes–Nash implementation when valuations are independent and signals are independently distributed with monotone hazard rate. Our goal is to study the problem for possibly interdependent valuations, for general joint distributions of signals, for ex-post and Bayes–Nash implementation, and for ex-post as well as interim individual rationality. It is too much to hope for closed form solutions in each case. Instead, the mechanism design problem is modeled as a combinatorial optimization problem, more precisely, as an integer linear program (ILP). The ILP has as set of feasible solutions the set of all mechanisms satisfying particular incentive and individual rationality constraints, and as objective the expected profit. The parameters of the constraints and the objective are determined by the set of possible signal profiles and their distribution. One of the challenges is to represent the expected profit as a linear function in the decision variables, the other is to provide, if possible, an efficient algorithm to solve the integer linear program. We are interested in polynomial-time algorithms in the number of agents and the number of signal profiles of the agents. We are also interested in gaining insights in economic properties of optimal mechanisms, and classes of instances where a "simple" mechanism is optimal.

To keep our model as general as possible we allow for informational externalities in the valuations[1] as well as for correlation in the signal distributions. The only restriction for the valuations is that for each agent they have to be strictly increasing in the agent's own signal. This condition together with the assumption of single-dimensional signals helps to avoid impossibility results regarding implementability (Jehiel et al., 2006). There has been a lot of literature on the role

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[1] We thank an anonymous referee to point out that our results in a previous version of this paper could be extended in this generality.

0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2013.03.014

of informational externalities since the seminal papers of Wilson (1967) and Milgrom and Weber (1982). An overview of major findings is provided by Jehiel and Moldovanu (2006). Branco (1996) studies profit-maximizing multi-unit auctions in the presence of informational externalities and provides regularity conditions under which the optimal mechanism can be characterized. For the welfare-maximizing public good problem Laffont and Martimort (2000) assume positively correlated signal distributions and they search for mechanisms that are strategy-proof against colluding coalitions.

Our main result is that the profit-maximizing, ex-post individual rational, ex-post incentive compatible mechanism can be computed in polynomial time. This holds for any number of agents, which is in sharp contrast to a recent result in Papadimitriou and Pierrakos (2011) on profit-maximizing single-item auctions for general distributions of signals. Papadimitriou and Pierrakos (2011) reduce the optimal auction problem to finding the maximum weight independent set on a k-partite graph, where k is the number of bidders. This yields a polynomial-time algorithm for the two bidder case. They further prove that the problem is NP-hard for more than two bidders. The former result is also provided independently by Dobzinski et al. (2011). Moreover, they develop a constant factor approximation for the case of an arbitrary number of bidders and an efficient algorithm to compute the optimal randomized mechanism that is truthful in expectation. We show that finding a profit-maximizing mechanism for the non-excludable public good problem for general distributions can be modeled as an integer linear program with a totally unimodular constraint matrix. More specifically, the ILP is equivalent to a maximal closure problem on partially ordered sets, where weights of the elements are equal to the sum of conditional virtual values minus the cost and the ordering is induced by the monotonicity constraints. Contrary to single-item auctions, this yields a polynomial-time algorithm for any number of agents. Notably, the optimal mechanism might choose to provide the public good for signal realizations where the sum of conditional virtual values is strictly smaller than the cost of providing the good. For the case of independent valuations and independent signal distributions with monotone hazard rate, we get the optimal Bayes–Nash implementable allocation rule of Güth and Hellwig (1986) as a byproduct.

For independent signal distributions and valuations without informational externalities, we illustrate, using recent results by Gershkov et al. (2013), that allowing for interim individual rationality instead of only for ex-post individual rationality or Bayes–Nash implementation instead of only for ex-post implementation cannot increase the expected profit. However, by virtue of examples it is shown that for interdependent valuations or for correlated signals, Bayes–Nash implementation may yield more profit than ex-post implementation. Transferring techniques developed for auctions by Crémer and McLean (1988) to the public good case, we illustrate that under some mild assumptions the profit-maximizing interim individual rational ex-post implementable mechanism can be determined in polynomial time. Furthermore as for auctions, it yields full surplus extraction. The same holds for Bayes–Nash implementation in the setting without informational externalities. By virtue of an example it is shown that full surplus extraction breaks down if we require ex-post individual rationality. It remains an open problem whether the optimal deterministic Bayes–Nash incentive compatible, ex-post or interim individual rational mechanism can be efficiently computed for interdependent valuations and general signal distributions.

2. Model and preliminaries

We are given a set of agents $N = \{1, \dots, n\}$ who hold private information, a signal $t^i \in T^i \subseteq \mathbb{R}$ about their values for consumption of a public good. We denote the set of signal tuples of all agents by $T := T_1 \times \cdots \times T_n$. All sets T^i are assumed to be finite. Signal tuples $t \in T$ occur according to the joint probability distribution φ and the cumulative distribution Φ. Furthermore, let φ_{t^i} be the probability that t^i occurs and Φ_{t^i} the corresponding cumulative distribution function.

The set of possible outcomes is $A := \{0, 1\}$, where 1 represents the event of providing the public good, while 0 means not providing it. For each $i \in N$ let $v^i := T \to \mathbb{R}$ denote i's valuation for a unit of the public good where it is assumed that $t^i \to v^i(t^i, t^{-i})$ is strictly increasing on T^i for each t^{-i}. Such valuations are called interdependent as the value of agent i depends not only on his own signal but also on other agents' signals. Settings with interdependent valuations are also said to have informational externalities. We normalize the value from not receiving the public good to zero. Agents act via a direct mechanism (f, p), which consists of an allocation rule $f : T \to A$ and a payment scheme $p : T \to \mathbb{R}^n$ as we allow for transfers. Agents are assumed to have quasi-linear utilities implying that for signal t^i and report profile $(s^i, t^{-i}) \in T$ the utility of agent i is $f(s^i, t^{-i})v^i(t^i, t^{-i}) - p^i(s^i, t^{-i})$.

The provision of the public good costs C. The profit $\pi(t)$ of the designer for a reported signal tuple t is the sum of payments collected minus the cost incurred:

$$\pi(t) = \sum_{i=1}^{n} p^i(t) - f(t)C.$$

We are seeking for mechanisms (f, p) that maximize the expected profit $\sum_t \varphi_t \pi(t)$. First, we search for the optimal mechanism among those that are ex-post incentive compatible and ex-post individually rational. Then we discuss other solution concepts. By the revelation principle the restriction to direct mechanisms is without loss of generality.

Definition 2.1. A mechanism (f, p) is ex-post incentive compatible (EPIC) if and only if for all agents i, for all fixed t^{-i} and for all $s^i \neq t^i$

$$f(t^i, t^{-i})v^i(t^i, t^{-i}) - p^i(t^i, t^{-i}) \geqslant f(s^i, t^{-i})v^i(t^i, t^{-i}) - p^i(s^i, t^{-i}).$$